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STRUCTURAL COMPUTER CODE EVALUATION

FINAL REPORT VOLUME I

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FOREWORD

The work described in this report was performed during fiscal year 1976. It was funded jointly by AF Rocket Propulsion Laboratory under Project Order No. 3059-76-1 (Job Order No. 305910MB) and by Strategic Systems Project Office under NWC Job Order No. 14571110. This effort was part of a task performed jointly by AFRPL and NWC to evaluate structural computer codes for analysis of rocket nozzles.

The work was documented in two volumes. Volume I contains a final report and summary of computer codes evaluated in this study; this work was performed mainly at NWC. Volume II contains results from solving several sample problems using selected computer codes, and this work was performed mainly at AFRPL.

This report is released at the working level for informational purposes only. Because of the continuing nature of structural computer code development, refinements and modifications may later be made in this study.

This report was reviewed for technical accuracy by Dr. W. J. Stronge, Code 6031, Naval Weapons Center.

This report has been reviewed by the Information Office/DOZ and is releasable to the National Technical Information Service (NTIS). At NTIS it will be available to the general public, including foreign nations. This report is unclassified and suitable for public release.

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NEPSAP	Nonlinear Structural Analysis	
NONSAP	Linear-Elastic Structural Analysis	
SAAS	Elastic-Plastic Structural Analysis	

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

This report summarizes the first phase of a study of nonlinear structural analysis computer codes for rocket nozzle stress analysis. A brief summary of nonlinear structural mechanics is given, along with a discussion of features desired for rocket nozzle thermomechanical stress analysis. Several computer codes are evaluated, and four codes are selected for further study in a subsequent phase of this work. Two of the selected codes employed linear elastic material behavior, while the remaining codes employed fully

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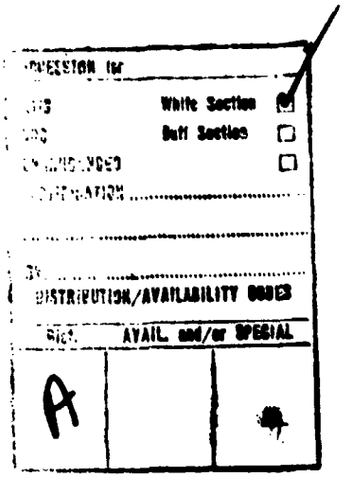
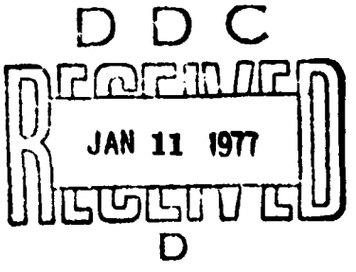
nonlinear behavior. The codes selected for this study were SAAS III, TEXGAP, NONSAP, and NEPSAP. It should be noted that none of the codes selected for this study contained all of the features desired for structural analysis of modern rocket nozzles; however, it is anticipated that future versions of these codes will contain the desired features.

In a subsequent phase of this work, the selected codes were evaluated and compared by solving sample problems similar to those found in the rocket nozzle industry. The results provided a useful baseline for establishing accuracy, computer run times, and ease-of-use of these codes. This work should be considered a continuing effort, since it is anticipated that further comparisons will be made as features are added to these codes or better codes become available.





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SECTION I

INTRODUCTION

Advanced nozzle materials are required for the severe thermo-mechanical environments produced by modern solid rocket motors. These materials are notable for their abilities to withstand extremely high temperatures ($> 5000^{\circ}F$) and retain adequate mechanical integrity for harsh loads such as those imposed by thermal stresses, internal pressures, TVC actuator forces, and dynamic effects. As rocket performance has increased, the demands made on nozzle components have correspondingly increased. Industry has responded by developing a multitude of new materials to meet these demands. Proliferation of these materials, many of which exhibit unique behavior under thermo-mechanical loading, has created a situation where our capabilities of characterizing the properties of these materials and predicting their structural performance are inadequate.

In recent years computer codes have been developed that can solve the generally nonlinear set of field equations that characterize advanced nozzle materials and geometries (e.g., see Ref. 1). Unfortunately, these codes are expensive to run, and they have not gained wide acceptance in the rocket propulsion community. Additionally, the ability of these codes to characterize the thermo-mechanical behavior of rocket nozzle materials, such as, 2-D and 3-D carbon fiber reinforced composites, has not been explored in great detail yet. Many of the newly developed codes are based on general continuum mechanics formulations of the field equations, i.e., kinematic, constitutive, and equilibrium relations, based on a given reference state; therefore, it is apparent that the essential elements for characterizing most materials are contained in these codes.

What presently remains is the application of these codes to specific rocket nozzle problems and the evaluation of their capabilities to model modern nozzle material behavior. Further work may also require the development of appropriate material property data or new material models to adequately characterize these advanced materials. Also, adequate failure criteria must be developed to cover the post-yield failure analysis of these materials.

The work presented herein is an initial step toward understanding and applying recently developed nonlinear analysis technology to the study of modern rocket motor materials. The ideas expressed herein were extracted from various sources, including conversations with personnel representing various DOD propulsion contractors and other agencies; such as Thiokol Chemical Corp., Hercules, Inc., United Technology - Chemical Systems Division, Aerojet Company, Lockheed Missiles and Space Company, Sunnyvale, and Sandia Corporation, Livermore. The author is solely responsible for the interpretation and translation of these discussions and for the opinions presented herein.

This work includes an evaluation of several computer codes and the application of these codes to some typical rocket motor structural problems. The codes range in complexity from linearly elastic, two-dimensional plane stress/plane strain and two-dimensional axisymmetric capability to completely general, nonlinear three-dimensional structure capability. These codes are representative of the current state-of-the-art in general stress analysis.

The work is presented in two volumes: the first of which contains a summary of the computer codes surveyed, a discussion of the criteria used to perform the preliminary evaluation of the codes, and a summary of sample problems used to evaluate the codes. A second volume will summarize the results from applying the selected codes to solve the sample problems.

Conclusions from this study include: (1) recommendations about the type of analytical features found desirable for use in rocket motor component stress analysis, (2) observations of the comparative ease-of-use and accuracy of the selected codes, and (3) identification of deficiencies in analytical modeling and material property characterization of the selected codes.

SECTION II

BACKGROUND

Several recent publications have surveyed state-of-the-art in computational methods of structural mechanics (Refs. 1, 2, and 3). Various papers from these references have formed the basis from which the present survey of structural analysis codes was obtained. The paper by W. A. Von Rieseman, et al, in Reference 1 was used as an initial screening device to obtain a list of candidate nonlinear computer codes for evaluation in this study. Additional codes were obtained from other sources and included in this list. User's manuals for the various candidate codes were acquired and studied prior to selecting four of these codes as being representative of current structural computing capability.

The codes that were chosen represent a range of computing capability from that which is currently being used in the propulsion community to that which is currently available for general nonlinear structural analysis. In terms of abilities, these codes solve problems ranging from linearly elastic two-dimensional, axisymmetric structures with orthotropic, temperature dependent properties to general, layered anisotropic three-dimensional structures with temperature dependent properties and various types of nonlinearities.

A portion of the present effort involved determining which nonlinearities would be significant in analyses of rocket motor components. In general, the types of nonlinearities in a structural system may be grouped into the following categories:

a. kinematic (geometric) nonlinearities - finite displacements, large strains.

b. constitutive (material) nonlinearities - creep, plasticity, viscoelasity, strain hardening, and others.

c. boundary condition nonlinearities - contact problems, crack propagation, ablation, and others.

d. thermal effects

e. buckling and instabil ty

f. nonconservative loadings

g. other effects

Most of the readily available computer codes are concerned only with the first four types of nonlinearities, and this report is mainly restricted to those areas, also. The following sections summarize in more detail the nonlinearities of interest in this study.

KINEMATIC NONLINEARITIES

The kinematic or strain-displacement relations for a continuum may be grouped into three categories, depending on which terms are retained in the strain-displacement relations. These categories are defined later, but first some preliminary definitions are given to help elucidate the following discussion.

In defining the displacement of a particle, one must determine both the original undeformed coordinates and the final deformed coordinates of the particle. Let the original undeformed coordinates be denoted by X^i (i=1, 2, 3) and the final deformed coordinates be denoted by x^i (i=1, 2, 3). The superscript is used to denote the contravariant components of these coordinates, while a subscript is used to denote the covariant components given by X_i and x_i. The coordinates X^i and X_i are called Lagrangian (or material) coordinates, while x^i and x_i are called Eulerian (or spatial) coordinates.

The components of displacement may then be expressed in terms of either contravariant or covariant components as:

$$u^i = x^i - X^i \tag{1}$$

or

$$u_i = x_i - X_i \tag{2}$$

When the strain-displacement relations are written in terms of Lagrangian coordinates in a general curvilinear coordinate system, the result defines the Green-Saint Venant strain tensor[1] (See Refs. 4, 5, 6, or 7 for details):

$$E_{ij} = \frac{1}{2}\left(u_i|_j + u_j|_i + u^m|_i\, u_m|_j\right) \tag{3}$$

[1] The notation employed herein follows that given in Refs. 4, 6.

where the vertical bar ($|$) denotes covariant differentiation with respect to the contravariant components (X^i) of the coordinates. That is,

$$u_i|_j = u_{i,j} - u_k \Gamma^k_{ij} \tag{4}$$

$$u^i|_j = u^i{}_{,j} + u^k \Gamma^i_{jk} \tag{5}$$

The symbols Γ^k_{ij} and Γ^i_{jk} denote Christoffel symbols of the second kind (see Ref. 4, for example); and the comma denotes ordinary partial differentiation. The Christoffel symbols are defined with respect to the Lagrangian coordinates X^i (i=1, 2, 3), and the ordinary partial derivatives are also taken with respect to the X^i. That is,

$$u_{i,j} = \frac{\partial u_i}{\partial X^j} \tag{6}$$

$$u^i{}_{,j} = \frac{\partial u^i}{\partial X^j} \tag{7}$$

As can be seen by the above relations, the development of the strain-displacement relations of a continuum in a general curvilinear coordinate system represents a complicated, albeit necessary, procedure. In order to reduce the complexity of the notation in this report and highlight only important elements of nonlinear mechanics, Cartesian tensors and coordinate systems will be employed in the remaining discussion; however, one should remain aware that many structural geometries are best described in curvilinear coordinate systems. Upon considering only rectilinear Cartesian coordinate systems, the Christoffel symbols vanish, and there is no distinction made between contravariant and covariant components. Then, the Green-Saint Venant Strain tensor takes on a much simpler form:

$$E_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i} + u_{m,i}\, u_{m,j}\right) \tag{8}$$

where the partial derivatives are defined by Equation 6.

The displacement gradient, $u_{i,j}$, may be expressed as the sum of a symmetric tensor, ϵ_{ij}, and an antisymmetric tensor, ω_{ij}.

Then,

$$u_{i,j} = \epsilon_{ij} + \omega_{ij} \tag{9}$$

where

$$\epsilon_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right) \quad \epsilon_{ji} \tag{10}$$

$$\omega_{ij} = \frac{1}{2}\left(u_{i,j} - u_{j,i}\right) = -\omega_{ji} \tag{11}$$

The quantities, ϵ_{ij} and ω_{ij}, are referred to as components of the infinitesimal strain and rotation tensors, respectively. The Green-Saint Venant strain tensor may then be expressed using the above definitions as:

$$E_{ij} = \epsilon_{ij} + \frac{1}{2}\left(\epsilon_{mi} + \omega_{mi}\right)\left(\epsilon_{mj} + \omega_{mj}\right) \tag{12}$$

The expressions given by Equation 8 and recast in Equation 12 represent the most general strain-displacement formulation available. The kinematic relations associated with these expressions may be described as defining LARGE (or FINITE) ROTATIONS and LARGE (or FINITE) STRAINS. They are also called FINITE DEFORMATION expressions.

When the elastic strains undergone by a body are small, ϵ_{ij} may be assumed to be an infinitesimal of the first order. If, in addition, the deformations (i.e., rotations) are allowed to remain large, then the strain-displacement relations may be approximated by the following expression:

$$E_{ij} \cong \epsilon_{ij} + \frac{1}{2}\,\omega_{mi}\,\omega_{mj} \tag{13}$$

Equation 13 represents the kinematic expression of LARGE ROTATIONS and SMALL STRAINS.

Finally, when both strains and rotations are assumed small (infinitesimals of the first order), the strain-displacement relations are given by

$$E_{ij} \cong \epsilon_{ij} \tag{14}$$

Equation 14 represents the kinematic relations found in the usual textbooks on "elasticity," and it corresponds to SMALL ROTATION, SMALL STRAIN behavior.

The Green-Saint Venant strain tensor is formulated with respect to the original undeformed coordinates of the body. It is also called the Lagrangian strain tensor. A Lagrangian formulation of the field equations is considered the most useful one for general nonlinear structural analysis (see paper by C. A. Felippa and P. Sharifi in Ref. 2).

An alternate formulation of the strain-displacement relations may be made based on the current coordinates of the deformed body. This alternate formulation is called an Eulerian description of the field equations, and it leads to a definition of the Almansi strain tensor given below.

$$e_{ij} = \frac{1}{2}\left(\underline{u}_{i,j} + \underline{u}_{j,i} - \underline{u}_{m,i}\,\underline{u}_{m,j}\right) \tag{15}$$

where the partial derivatives are now taken with respect to the deformed configuration. That is, $\underline{u}_{i,j} = \partial u_i/\partial \underline{x}^j$, and the bar below the variable ($\underline{\ }$) is used to signify that fact.

Equation 15 may also be expressed in terms of a symmetric tensor, $\underline{\epsilon}_{ij}$, and an antisymmetric tensor, $\underline{\omega}_{ij}$, as before; but now referenced to the deformed configuration.

$$e_{ij} = \underline{\epsilon}_{ij} - \frac{1}{2}\left(\underline{\epsilon}_{mi} - \underline{\omega}_{mi}\right)\left(\underline{\epsilon}_{mj} - \underline{\omega}_{mj}\right) \tag{16}$$

where

$$\underline{\epsilon}_{ij} = \frac{1}{2}\left(\underline{u}_{i,j} + \underline{u}_{j,i}\right) = \underline{\epsilon}_{ji} \tag{17}$$

$$\underline{\omega}_{ij} = \frac{1}{2}\left(\underline{u}_{i,j} - \underline{u}_{j,i}\right) = -\underline{\omega}_{ji} \tag{18}$$

Approximations can be also made in Equation 16, as before, leading to LARGE ROTATION, SMALL STRAIN or SMALL ROTATION, SMALL STRAIN behavior.

It should be noted that both the Lagrangian and Eulerian strain-displacement formulations lead to symmetric strain tensors. That is,

$$E_{ij} = E_{ji} \tag{19}$$

and

$$e_{ij} = e_{ji} \tag{20}$$

For infinitesimal strains and rotations there is no need to distinguish between E_{ij}, e_{ij}, ϵ_{ij}, or $\underline{\epsilon}_{ij}$. However for large strain, large rotation problems, the distinctions must be noted and properly accounted for. These distinctions are particularly important when specifying input material properties as will be discussed in following sections.

CONSTITUTIVE (MATERIAL) NONLINEARITIES

In describing constitutive laws, one must first define the state of stress through which the laws are related to the state of strain. There are two descriptions of stress that are commonly employed; however, additional states of stress have been defined (see Ref. 6). The usual descriptions of stress are the Cauchy stresses, σ_{ij}, defined in an Eulerian coordinate systems and the second Piola-Kirchoff stresses, S_{ij}, defined in a Lagrangian coordinate system.

The Cauchy stresses are defined as the current forces acting on the current areas of a deformed body, while the second Piola-Kirchoff stresses are defined as forces transformed to the original undeformed coordinate system acting on areas of the undeformed configuration. These two definitions of stress lead to symmetric stress tensors while other definitions (such as, the first Piola-Kirchoff stresses) do not. Additionally, these two stresses are completely complementary with the previous definitions of strain. That is, Cauchy stresses

are complementary with Almansi strains, while second Piola-Kirchoff stresses are complementary with Green-Saint Venant Strains. The symmetry of the stress and strain tensors lead to simplifications of the constitutive laws that are highly desirable, reducing the number of unknowns in the case of general anisotropic elasticity from 81 constants to only 21 (which is still quite a few). These twenty-one constants may be reduced to fewer unknowns when various symmetries are present in a material.

The constitutive relations in nonlinear continua fall into two broad classes that may be called "elastic materials" and "inelastic materials." The classes of "elastic materials" are fairly well defined on axiomatic foundations yielding three subclasses of behavior: elastic, hyperelastic, and hypoelastic behavior (see Ref. 6). Inelastic materials have not been so well defined.

Elastic Materials

Elastic materials are described as those in which there is a one-to-one correspondence between Cauchy stresses and Almansi strains. The mechanical properties then depend only on the current deformation or state of strain (see Refs. 5, 6, 7), but not on the strain histories. This class of materials includes both linear and nonlinear elastic behavior.

The stress-strain relations for elastic materials may be written in terms of Cauchy stresses and Almansi strains as

$$\sigma_{ij} = C_{ijk\ell} e_{k\ell} \tag{21}$$

A similar expression may be written for second Piola-Kirchoff stresses and Green-Saint Venant strains.

$$S_{ij} = C_{ijk\ell} E_{k\ell} \tag{22}$$

where the stresses and strains in the two different coordinate systems are related by the following expressions.

$$S_{ij} = \frac{\rho_o}{\rho} \frac{\partial X_i}{\partial x_\alpha} \frac{\partial X_j}{\partial x_\beta} \sigma_{\alpha\beta} \tag{23}$$

and

$$E_{kl} = \frac{\partial x_\gamma}{\partial X_k} \frac{\partial x_\delta}{\partial X_l} e_{\gamma\delta} \tag{24}$$

The bar (_) under the elastic constants in Equation 21 points out that these constants are defined in terms of the deformed or Eulerian coordinates, while in Equation 22 they are defined in terms of the original undeformed or Lagrangian coordinates. This feature of nonlinear analysis raises an important issue that was mentioned in the previous section. That is, in specifying input material properties, the analyst must know which ones are employed in a particular computer code, and he must have taken steps to make his input data consistent with the analysis being performed. This is no problem for elastic materials since the properties are related by the simple coordinate transformations given below.

$$C_{\alpha\beta\gamma\delta} = \frac{\rho_o}{\rho} \frac{\partial X_\alpha}{\partial x_i} \frac{\partial X_\beta}{\partial x_j} \frac{\partial X_\gamma}{\partial x_k} \frac{\partial X_\delta}{\partial x_l} \underline{C}_{ijkl} \tag{25}$$

or

$$\underline{C}_{ijkl} = \frac{\rho}{\rho_o} \frac{\partial x_i}{\partial X_\alpha} \frac{\partial x_j}{\partial X_\beta} \frac{\partial x_k}{\partial X_\gamma} \frac{\partial x_l}{\partial X_\delta} C_{\alpha\beta\gamma\delta} \tag{26}$$

where $\frac{\rho}{\rho_o} = \text{determinant} \left| \frac{\partial X_i}{\partial x_j} \right|$

ρ = density of the material in the deformed configuration

ρ_o = density of the material in the undeformed configuration

A few of the computer codes examined in this study (e.g., GNATS, NONSAP) allow options on the choice of coordinate systems employed to input the data. It is also possible to mix coordinate systems and arrive at still further descriptions of constitutive relations between stresses and strains. This aspect of large deformation theory has particular significance when typical uniaxial test data is employed in an analysis. Usually, uniaxial test data is plotted as stress (force divided by original area) versus engineering strain (change in length divided by original length). The stress in this case is equivalent to the second Piola-Kirchoff stress, since the orientation of the force vector is

the same in both the deformed and undeformed coordinate systems. The strains measured by the engineering definition are different from either Green-Saint Venant or Almansi strains, as is shown in the following table (borrowed from Ref. 8).

Table 1. Comparison of Strain Measures for Uniaxial Strain

Engineering Strain - $\Delta l/l_o$	0.02	0.05	0.1	0.5
Green Strain - $E_{11} = \frac{1}{2}\left[(l/l_o)^2 - 1\right]$	0.0202	0.0513	0.105	0.625
Almansi Strain - $e_{11} = \frac{1}{2}\left[1 - (l_o/l)^2\right]$	0.0194	0.0465	0.0868	0.278

Obviously, the elastic constants defined using engineering strain are not compatible with those required for large deformation analysis unless appropriate coordinate transformations have been made or the strains are small. The results shown in Table 1 indicate that when the engineering strains are 2 percent ($\Delta l/l_o = 0.02$) or less, there are no significant differences between the Engineering, Green, or Almansi strains. Then, small strains can be defined as those that occur at 2 percent strain or less.

A potential problem that could arise from using input data defined in the wrong coordinate system is that one might be lead to erroneous conclusions concerning the calculated stress and strain levels. This could be especially embarrassing if one compared a stress computed in one coordinate system to a design allowable defined in another system, and then predicted positive margins of safety when they were really negative.

The problem of determining appropriate constitutive constants is reduced when strains are small. In this case, the differences between the coordinate systems are negligible, and the material constants may be interchanged without much error. Results such as those given in Table 1 can be used to determine how much error is involved for various levels of strain. A special class of nonlinear elastic behavior is defined when strains are small. This class is called material nonlinear elastic behavior, and it is possible to solve this class of problems with a linear elastic model provided appropriate material constants are used. In general, an "equivalent" set of elastic material constants must be determined such that the kinematic, kinetic (or equilibrium), and constitutive

relations are satisfied simultaneously. It is usually necessary to do this iteratively for each stress-strain state desired. In order to perform this type of computation, one must know all of the nonlinear material constants for the full range of stress and strain desired. Material models in this group of problems may include multilinear segment and curve description models.

It should be pointed out that material nonlinear behavior is not the same as elastic-plastic behavior. Elastic-plastic behavior is more complicated since not only elastic behavior, but yield conditions, flow rules, and hardening rules are all involved in an elastic-plastic material description. It is true, however, that solution algorithms for nonlinear elastic and elastic-plastic behavior may employ the same computational features.

Hyperelastic Materials

These materials are characterized as possessing a strain energy function per unit mass, ξ. The strain energy is an analytic function of the strain tensor formed with respect to the stress free state, such that the rate of change of strain energy is equal to the rate of work done by the stresses. That is,

$$\frac{D\xi}{Dt} = \frac{1}{\rho} \sigma_{ij} \frac{De_{ij}}{Dt} \tag{27}$$

where D/Dt is the well-known material derivative. Alternately, Equation 27 may be expressed as (see Ref. 5),

$$\sigma_{ij} = \frac{1}{2} \rho \left(\frac{\partial \xi}{\partial e_{ij}} + \frac{\partial \xi}{\partial e_{ji}} \right) \tag{28}$$

Many material models that fit this class of behavior have been proposed. In general, the models are applicable to both isothermal and isentropic processes. The strain energy function is sometimes expressed as an elastic potential measured per unit volume of the undeformed configuration (Ref. 5) as shown below.

$$W = \rho_o \xi \tag{29}$$

then

$$\sigma_{ij} = \frac{1}{2}\frac{\rho}{\rho_o}\left(\frac{\partial W}{\partial e_{ij}} + \frac{\partial W}{\partial e_{ji}}\right) \tag{30}$$

The elastic potential is usually assumed to be an analytic function of the strain tensor, e_{ij}, so it may be expanded in series form as given below.

$$W\left(e_{ij}\right) = \sum_{i=1}^{\infty} W_i\left(e_{ij}\right) \tag{31}$$

where $W_i\left(e_{ij}\right)$ is expressed as an i^{th} degree, homogeneous polynomial in e_{ij}. Then the elastic potential may be written as:

$$W\left(e_{ij}\right) = W_o + \underline{C}^{ij}e_{ij} + \frac{1}{2}\underline{C}^{ijk\ell}e_{ij}e_{k\ell} + \frac{1}{3}\underline{C}^{ijk\ell mn}e_{ij}e_{k\ell}e_{mn} + \cdots \tag{32}$$

The coefficients, $\underline{C}^{ij}$, $\underline{C}^{ijk\ell}$, $\underline{C}^{ijk\ell mn}$, ..., are "elasticities" of various orders and W_o is an arbitrary constant. The "elasticities" form the material constants that must be evaluated to characterize a particular material. Since the components of the strain tensor may be expressed in terms of certain invariants of the tensor (e.g., see Ref. 5), the elastic potential may also be expressed in terms of these invariants. It can be shown that the "elasticities" in Equation 32 are all symmetric with respect to each successive pair of indicies.

Various material models have been constructed based on Equation 32, and many of these models are described in Reference 5. A particular model is developed when the initial state is stress free (i.e., $\underline{C}^{ij} = 0$) and small deformations are assumed. Then, Equation 32 reduces to that of classical linear elasticity.

$$W\left(e_{ij}\right) \equiv W\left(\underline{\epsilon}_{ij}\right) = \frac{1}{2}\underline{C}^{ijk\ell}\underline{\epsilon}_{ij}\underline{\epsilon}_{k\ell} \tag{33}$$

Additional material models in this class include isotropic, orthotropic, anisotropic, and incompressible materials. Since hyperelasticity includes incompressible materials, such as rubber, it is commonly used to denote only rubbery-type behavior, although it has been seen that various "elastic" materials are included in this definition of material behavior. A well known example of a

rubber material model in this class is the Mooney-Rivlin model for large strains. Other examples are given in Reference 5.

Hypoelasticity

A hypoelastic material is one in which the components of stress rate are homogeneous linear functions of the deformation rate. That is,

$$\sigma_{ij}^{\nabla} = \underline{C}^{ijkl} \dot{e}_{ij} \tag{34}$$

where σ_{ij}^{∇} is the rate of stress and the dot $(\cdot)$ denotes time differentiation. It should be observed that the stress rate in Equation 34 must be defined so that it is invariant under rigid-body rotations of the material. Several definitions of this stress rate have been made (e.g., see Ref. 6), since it is not unique. The Jaumann stress rate given below is employed in the computer code NONSAP.

$$\sigma_{ij}^{\nabla} = \frac{D\sigma_{ij}}{Dt} - \sigma_{ip}\underline{\Omega}_{pj} - \sigma_{jp}\underline{\Omega}_{pi} \tag{35}$$

where $\underline{\Omega}_{ij}$ are the components of the spin tensor. That is, $\underline{\Omega}_{ij}dt = \underline{\omega}_{ij}$, the rotation tensor.

Although the theory of hypoelasticity is well formulated and has potential application, according to Eringen (Ref. 7) there are no known solids that exhibit hypoelastic behavior.

Inelastic Materials

The status of inelastic materials is much less well defined than that of elastic materials. This class of materials includes those with memory; such as viscoelastic and creep materials, along with those whose response is path dependent, such as, elastic-plastic materials. It is interesting to note that most of the constitutive models developed in this class of materials pertain only to small strains where elastic and plastic strains are the same order of magnitude, although large rotations may be allowed. This fact was pointed out for elastic-plastic materials by Lee in Reference 9. He indicated that in most theories it is assumed that elastic and plastic strains may be superimposed to obtain total strain; however, this superposition is not valid when strains are large. Lee

proceeded to develop a constitutive theory for large strain, large rotation elasto-plasticity in that reference. Other investigators have addressed the large strain plasticity problem; however, in general, these theories have not yet been well enough developed to implement into finite element computer codes.

Elastic-Plastic Materials

As mentioned earlier elastic-plastic materials are characterized by their elastic behavior, a yield condition, a flow rule, and a hardening rule. The elastic behavior may be nonlinear as previously described, but most theories assume linear elastic behavior. There are too many theories of plastic material behavior to discuss in detail in this report. A concise description of many of the theories is given in the article by H. Armen in Reference 1; also, see Reference 6.

The theories of plasticity are grouped into two broad classifications: DEFORMATION (or Hencky) theory and INCREMENTAL (or flow) theory. In deformation theory, it is assumed that the state of stress and the total strain are uniquely related (i.e., a one-to-one correspondence between stress state and total strain state). In this sense, deformation plasticity theory is similar to nonlinear elastic theory. In incremental theory, on the other hand, it is assumed that the final state of strain depends on the history of loading; that is, increments of the plastic strains are related to the stress state, and the final strain state depends on the load path taken.

A YIELD CRITERION is used to distinguish the point of departure from elastic behavior in both of these theories. The phenomenon of yielding distinguishes the plasticity theories from nonlinear elastic theory. Many yield criteria have been proposed to model various types of material behavior observed. The most commonly used criteria are the Tresca-Mohr and Hencky-Von Mises criteria for isotropic materials and the Hill-modified Von Mises Criterion for anisotropic materials. There have been several modifications of these criteria to fit particular materials. These modifications are usually made to include the effects of the dilatational (or volumetric) stress components that are neglected in the above criteria. Additional yield criteria, attributed to Coulomb-Mohr and Ducker-Prager, have been devised to account for cohesive and frictional strengths, respectively.

Material behavior after yielding is governed by a FLOW rule in incremental theory and by the appropriate stress-strain law in deformation theory. The flow rule describes how the increments of plastic strain are related to the state of stress.

Most of the well known plasticity models apply only to isotropic material behavior, and are therefore most appropriate for metallic structures. Some of the better known plasticity models are:

a. RIGID, PERFECTLY PLASTIC - the elastic response is completely neglected, and the plastic stress-strain curve is constant and equal to to the yield stress. The complementary flow rule for this model is attributed to Levy-Mises.

b. ELASTIC, PERFECTLY PLASTIC - the elastic response is included and superimposed on the plastic strains. The plastic portion of the stress-strain curve is constant, and the flow rule is attributed to Prandtl-Reuss.

c. ELASTIC-STRAIN HARDENING - this material is characterized by a bilinear stress-strain curve; the formulation of a hardening rule is often based on the concept of equivalent (or effective) stress and strain. The Prandtl-Reuss flow rule is often used; however, other flow rules have been developed based on a plastic potential function concept.

The behavior of a material during flow is governed by the HARDENING rule. Several hardening rules are available, but the most common ones are isotropic hardening, kinematic hardening, the Mroz model of hardening, and the mechanical sublayer model of hardening. An excellent discussion of these and other hardening theories is given in Reference 10.

It should be reiterated that all of the above well-known plasticity theories concern only isotropic materials undergoing small strains. Extensions of these theories have been made to anisotropic materials (e.g., see Ref. 11); however, few anisotropic theories are available in current finite element computer codes.

Viscoelastic Materials

Viscoelastic materials are characterized by behavior that depends on the entire history of stress and strain. The constitutive laws for these materials may be generally expressed in terms of rate operators or hereditary integrals involving time. Present theories are usually restricted to linear viscoelastic behavior where the superposition principle holds with respect to the differential and integral operators employed in the material models. Also these material

models are restricted to small strains, for the same reasons given in the discussion on plasticity.

A general, small strain thermoviscoelastic constitutive law may be written as

$$\sigma_{ij} = \int_{-\infty}^{t} \underline{C}_{ijkl}\left(\xi - \xi'\right) \frac{\partial}{\partial \tau}\left(\varepsilon_{kl} - \alpha_{kl}\theta\right) d\tau \tag{36}$$

where $\underline{C}_{ijkl}$ are 21 anisotropic relaxation moduli, and thermal strains are included with the $\alpha_{kl}\theta$ term. The variable ξ is called the "reduced time" and it is given by

$$\xi\left(x_i, t\right) = \int_{o}^{t} \frac{d\tau}{a\left[T\left(x_i, t\right)\right]} \tag{37}$$

where a(T) is an experimental time shift function determined solely in terms of the temperature, $T\left(x_i, t\right)$.

The above expressions define THERMORHEOLOGICALLY SIMPLE material behavior. The relaxation moduli, $\underline{C}_{ijkl}$, form the kernels of the hereditary integral in Equation 36, and these functions must usually be determined experimentally. Nonlinearities are included in the above expressions by application of the reduced time concept and the shift function, a(T).

Creep Material Behavior

In general, creep behavior lumps all time dependent material flow into one category; however, in practice, creep is usually used to specify long term material flow. The creep time frame is separated for analysis purposes into three regions; primary creep (initial, decreasing creep rate), secondary creep (steady creep rate), and tertiary creep (final, increasing creep rate). Although there is much similarity between creep, viscoelasticity, and plastic flow, the analytical models employed may be quite different. A variety of creep models have been proposed to exploit the characteristics of different materials. These models include hereditary integrals, exponential laws, and power law representations. Generally, creep laws exhibit a dependence of material properties on

time, temperature, and stress levels. An example of a hereditary creep law is given below.

$$\varepsilon_{ij}(t) = \int_{-\infty}^{t} J_{ijkl}\left(\sigma_{kl},\ t-\tau\right)\frac{\partial \sigma_{kl}}{\partial \tau}\,d\tau \tag{38}$$

where the J_{ijkl} are called creep compliance functions; and for a particular material they are mathematically related to the stress relaxation moduli.

BOUNDARY CONDITION NONLINEARITIES

Nonlinear boundary conditions may involve forces or displacements, or both. An example of a nonlinear force boundary condition is the so-called FOLLOWER FORCE. In this example, the applied force moves in such a manner that it is always normal to the surface on which it is applied. Other examples include forces that are nonlinear functions of displacement; such as hardening or softening springs, and contact forces between bodies. The latter nonlinear behavior is distinguished from the first two examples in that the forces are nonlinear functions of the changing area of contact between the bodies. Examples of nonlinear displacement boundary conditions are sliding contact between two bodies and crack propagation.

Obviously this list could be extended further, but these examples serve to illustrate some types of nonlinear boundary conditions that may be encountered.

THERMAL NONLINEARITIES

Thermomechanical behavior of solids is distinguished from solely mechanical behavior since, in general, the equations of heat conduction and mechanical behavior are coupled. In many engineering problems, however, it is possible to neglect the coupling effect; and an uncoupled thermomechanical theory results.

The simplest form of uncoupled theromechanical behavior is the theory given by the Duhamel-Neumann law:

$$\sigma_{ij} = C_{ijkl}\,e_{kl} - \beta_{ij}\,\theta \tag{39}$$

where the β_{ij} are the thermal moduli (related to the coefficients of thermal expansion) and θ is the temperature difference with respect to a reference

temperature. The elastic moduli, C_{ijkl}, are similar to those defined in Equation 21 except that they are allowed to depend also on temperature.

The temperature distribution, $\theta(x_i, t)$, is given by the usual, uncoupled heat conduction equation. In general, the uncoupled temperature analysis involves nonlinear procedures.

In the thermoelastic theory, large deformations may be allowed and the elastic material constants may be nonlinear.

Extensions of the uncoupled theory to include thermoplastic behavior follows an approach similar to that described above. Thermoviscoelastic behavior has been described in a previous section.

Material constants do not necessarily behave linearly with respect to temperature in the above cases; however, there must be a one-to-one correspondence between the material constant, the temperature, and the state of strain.

There are many effects due to time, strain rate, heating rate, and others that are not included in these models. Obviously, there is still much work needed in these areas of material characterization to describe real material behavior.

SECTION III

COMPUTER CODE EVALUATION CRITERIA

An evaluation of nonlinear computer codes for use in solid rocket nozzle stress analyses requires many judgements to be made. Some of these judgements are subjective, such as, deciding which computer codes are the easiest to learn and use; others are objective, such as determining which of the selected codes give the best answers with the lowest run times. Since many of the codes evaluated herein are constantly being updated, it is likely that the conclusions of this study will be revised at some time. It is also apparent that there are other significant problems, besides structural analyses and stress predictions, that must be considered in rocket nozzle design (e.g., see Ref. 12). It is intended that these additional problems will be addressed in follow-on studies.

The first step in the current evaluation process was a screening of the candidate computer codes to determine the adequacy of the codes to model the desired structural and material behavior. It is fair to say that none of the candidate codes had all of the structural modeling features desired. Therefore, some compromises were made in selecting the codes, and one level of subjectivity was introduced. In general, the features desired in the computer codes were:

a. FINITE ELEMENT FORMULATION - Only finite element computer codes were evaluated because of their well-known advantages in treating complex, multicomponent parts with complicated boundary conditions.

b. AXISYMMETRIC GEOMETRIES - Rocket nozzles are axisymmetric bodies composed of many different pieces and materials. This type of geometry can be modeled with 2-dimensional axisymmetric ring elements or with general 3-dimensional solid elements.

c. AXISYMMETRIC AND NONAXISYMMETRIC LOADING - Most often the thermomechanical loading on a nozzle can be considered axisymmetric; however, there are important instances when nonaxisymmetric loads and material characteristics must be considered.

d. ANISOTROPIC MATERIAL CAPABILITY - Many rocket nozzle materials may be considered axisymmetric orthotropic or transversely isotropic; however, the advanced materials, such as, carbon/carbon rosette layups must be considered anisotropic. Additionally, nonaxisymmetric temperature distributions cause material variations around the circumference which require nonaxisymmetric, rather than axisymmetric, treatment.

e. MATERIAL AND GEOMETRIC NONLINEARITIES - It is well known that rocket nozzle materials exhibit nonlinear, small strain behavior over the wide range of temperatures in which they are required to operate. It is also likely that plastic deformations take place in addition to the nonlinear behavior, and these plastic deformations may result in large strains. Geometrically nonlinear deformations due to large rotations may also occur on large diameter, thin shell structures, even though the material is undergoing small elastic strains.

f. TEMPERATURE DEPENDENT PROPERTIES - The temperature dependence of rocket nozzle materials is well documented. It is important that this characteristic be accurately assembled into a computer code, since stresses induced by thermal expansions can be many times larger than those due to purely mechanical loadings.

g. VARIETY OF MATERIAL MODELS - Rocket nozzles are composed of many types of materials: metals, graphites, phenolics, carbon-fiber reinforced composities, and others. Obviously, a lot of different material models, such as those described earlier in this report, are required to model this variety of materials.

h. NONPROPRIETARY CODES - A very important consideration in this evaluation was that any codes chosen should be non-proprietary. There are a few good proprietary nonlinear codes available, but they were excluded from consideration in this study. The reasons for this exclusion included initial costs and a lack of knowledge of the inner workings of the codes. It was considered desirable that the programs selected be in a form which could be modified in-house, with minimum technical assistance available from the author of the code.

i. ESTABLISHMENT OF BASELINE CODE FOR COMPARISONS - It was considered necessary to establish a baseline computer code for comparing various codes being evaluated. It was felt that the baseline code should exhibit the following characteristics:

   (1) It should be well documented and in use throughout the propulsion industry.

   (2) It should exhibit most of the basic features considered desirable for nozzle stress analysis; i.e., thermal stress computations, axisymmetric geometries, variety of materials and material models, and etc.

   (3) It should have a well-established record of stress analysis computations, and have generated a high degree of user confidence in its application.

   (4) The baseline code should perform linear elastic analyses and be representative of the current computing capability available in the propulsion industry.

   (5) By comparing the other codes being evaluated to the baseline code, one should be able to draw conclusions about relative efficiencies, relative accuracies, ease of use, advantages, disadvantages, and etc.

The SAAS III computer code (Ref. 13) was selected as the baseline code. since it is currently in wide use for thermo-mechanical nozzle analysis.

j. OTHER FACTORS - Several other factors also influenced the selection of codes in this study. These factors fall in what might be classified as a nice-to-have category, and they are summarized below.

(1) Is the program complete and well documented? Does the documentation contain a complete theoretical description, or is it basically a user's manual?

(2) Is the code available in different machine configurations, and how much of the coding is system dependent?

(3) Is the code modularized in concept so that it may be expanded or added to at the user's option?

(4) Does the code employ efficient and accurate computational algorithms?

(5) Are the material models well founded on theoretical bases, and are the restrictions of the models clearly defined?

(6) Does the code employ the latest technology in finite element models, such as, isoparametric elements, etc?

(7) Are there any special features dealing with material modeling, boundary conditions, or input data employed in the codes?

(8) Are pre- and postprocessors available for the code?

(9) Does the code have companion thermal analyzers, or is it necessary to interpolate thermal data from other sources?

All of the above factors were considered to some extent in the selection of codes used in this study. The next section of the report describes the features of four codes selected for further evaluation, while a summary of the remaining codes evaluated is given in Appendix A.

SECTION IV

SUMMARY OF SELECTED CODES

It was apparent on review of all the user's manuals acquired for this study that no single computer code contained all of the attributes necessary for general nonlinear nozzle structural analysis. In many cases the main deficiency was lack of a material model to solve anisotropic, elastic-plastic materials. Notwithstanding the fact that obtaining material properties for anisotropic, elastic-plastic materials is a complex task, it was frustrating that there have been few if any adequate material models developed to properly characterize this material behavior commonly found in nozzle structures. Because this situation exists, the task of selecting computer codes for nozzle analyses becomes one of choosing codes with the fewest deficiencies and assuming that the answers produced by such codes can be used with engineering judgement to design nozzles.

The computer codes chosen for this study provide a summary of capabilities available in the structural analysis community. These codes fairly well depict the current state-of-the-art in structural analysis, but they do not represent the ultimate code for which we are looking to perform nozzle structural analyses. It is believed that a variety of computational tools will be needed to perform nozzle structural analyses. It is desireable to have simple tools to perform simple analyses and to have complex tools for complex analyses when they are required. The selected codes represent a range from relatively simple to fairly complex structural analysis. As much as possible, the codes were selected on the basis of the desired features discussed in the previous section.

The selected codes are described below along with the primary reasons for their selection. Descriptions of the selected codes are given in following sections, and descriptions of the remaining codes evaluated are given in Appendix A. The codes chosen for further evaluation in this study were SAAS III (Ref. 13), TEXGAP (Ref. 14), NEPSAP (Refs. 15, 16), and NONSAP (Refs. 17, 18).

SAAS III was chosen as the baseline code with which the other codes were to be compared. It was chosen on the basis that either SAAS III or a code like it (e.g., AXISOLV written by Dr. E. L. Wilson, Univ. of California, Berkeley,

or AMG-049 written by J. J. Brisbane of the Rohm and Haas Company) was available and familiar to most stress analysts in the propulsion community. SAAS III represents fairly old finite element technology, but it is reliable and has the capability for solving a wide variety of axisymmetric, thermo-mechanical stress problems. By comparing results obtained from the other selected codes with those obtained from SAAS III, one should be able to form opinions and conclusions about the other codes.

Although SAAS III contains a bilinear material model for nonlinear behavior, it is basically a linear elastic computer code. The bilinear approximation is based on an "equivalent" stress-strain approach and a deformation plasticity theory. The code employs low order (constant strain type) finite elements, and an iteration procedure based on "equivalent" elastic constants is used to determine the nonlinear stress solution.

The TEXGAP computer code was selected for comparison in this study because it represents an improvement in linear elastic finite element modeling over the SAAS III code. The main improvements are in the use of higher order (isoparametric type) finite elements and in the application of the wave front equation solution technique. TEXGAP employs many special features, such as, a cracked finite element, an incompressible finite element, and a rezoning feature. These features were of secondary interest in the current study; however, they make the code very attractive for anyone interested in two-dimensional, axisymmetric analysis.

The NEPSAP computer code represents a large, general purpose nonlinear structural analysis code. It includes both material and large displacement nonlinearities, along with a fairly complete finite element library. Because NEPSAP is a very general computer code, it was desired to assess the penalties (if any) for using a general code as opposed to using the special codes, SAAS III and TEXGAP. Since the first solution step of a nonlinear solution is the linear step, we wanted to see what the differences in linear elastic analysis might be between the special purpose codes and a general code. Additionally, the general code has the capability for extending a solution to fully nonlinear behavior with three-dimensional, nonaxisymmetric geometries, if it becomes necessary.

The version of NEPSAP employed in this study was a nonproprietary, static solution code that was available from ASIAC at Wright Patterson AFB.

An improved proprietary version that includes nonlinear dynamic and modal analysis capability is currently being debugged at LMSC, Sunnyvale. According to LMSC personnel, the newer version contains an increased finite element library and more efficient solution algorithms. This code is designed as a production code and has extensive pre- and postprocessing available.

NONSAP was selected for evaluation in this study because it contains the most advanced material models and finite element formulations of any of the codes considered. It must be considered a general purpose code that is capable of performing fully three-dimensional, nonlinear structural analysis; although only the two-dimensional and axisymmetric finite elements contain a fairly complete set of material models. This code is the only one of the four selected codes that presently has dynamic analysis capability. It is considered by its authors to be mainly a research tool. That is, it was not intended for production stress analyses, and therefore the solution algorithms were not necessarily optimized for maximum efficiency.

In order to obtain a valid comparison of these codes, it was decided that they should be compiled and executed on the same computer system. The AFRPL CDC 6400 was chosen as the machine on which the comparison was to be made.

A detailed description of each of the selected codes is given in the following sections.

SAAS III

This code was written by J. G. Crose and R. M. Jones (Ref. 13), and it is the latest version in a series of codes that are based on the original work of Dr. E. L. Wilson, University of California, Berkeley. The code is restricted to two-dimensional, axisymmetric solids with axisymmetric mechanical and thermal loading. This code is not as general as one would like for the analysis of anisotropic nozzle materials, but it is efficient and widely used as a design tool. The code will perform only static stress analyses.

Elements

A. Current element library

1. Constant strain triangle (CST) - A three node (6DOF-2DOF/node) triangular element based on linear displacement functions is utilized.

2. Quadrilateral element - A four node (8DOF-2DOF/node) quadrilateral element composed of four constant strain triangular elements is available.

B. Notes

1. The elements may be used for either plane stress/plane strain or axisymmetric solids of revolution.
2. Axisymmetric solids are restricted to axisymmetric kinematic behavior, material properties, and loadings.

Material Behavior

A. Current material models

1. Isotropic and orthotropic (axisymmetric) thermoelastic behavior is considered.
2. Thermoelastic-plastic models with axisymmetric, orthotropic elastic-plastic behavior are used. A modified Von Mises yield criterion and a deformation plasticity theory are employed.
3. Porous elastic material behavior is considered.
4. Unequal properties in tension and compression are allowed in the thermoelastic model.

B. Notes

1. The thermoelastic and thermoelastic-plastic behavior are based on uncoupled theories.
2. The porous-elastic behavior is uncoupled.
3. Orthotropic bilinear (elastic-strain hardening type) elastic-plastic behavior is employed. The model is based on a "normalized effective stress-strain" approach. This model should be carefully examined for its applicability to any material being studied.

Loading Conditions

A. Current loading conditions allowed

1. Boundary pressure loads and boundary shear stresses may be applied.
2. Thermal loading may be applied.
3. Pore pressures (for porous materials) may be applied.
4. Body forces due to angular rotation and axial acceleration may be applied to axisymmetric bodies while linear acceleration may be applied to plane stress/plane strain structures.
5. Concentrated nodal loads are allowed.

B. Notes

1. Skewed nodal concentrated loads may be applied.
2. Temperatures are linearly interpolated if the input temperature locations do not coincide with nodal points.
3. Only axisymmetric loads are allowed.

Boundary Conditions

A. Current boundary conditions allowed

1. Nodal displacements may be specified.
2. Skewed displacement boundary conditions are allowed.

B. Notes

1. Nonzero boundary displacements may be specified.

Solution of Equations

A. Solution method

1. Gaussian elimination is applied to a symmetric, banded matrix.
2. Elastic-plastic solutions are obtained using successive approximations, and the stiffness matrix is updated at each iteration.

B. Notes

1. A constant bandwidth is assumed.
2. A maximum of 1000 nodal points is allowed, leading to a maximum of 2000 equations that can be solved.

Special Features

A. Preprocessor

1. Mesh generation is available.
2. A Laplacian mapping scheme is employed in the mesh generation.

B. Postprocessing

1. Plot packages are available for various machines and plotters, including CALCOMP and FR80.

C. The SAAS III computer code is operational on IBM 360/65, UNIVAC 1108/1110, and CDC 6600 machines.

Comments

The SAAS III computer code appears to be ideally suited for linear elastic analyses of many types of nozzle structures. It should provide a good assessment of many design features since it can handle a variety of material types, loading conditions, and boundary conditions. The solution algorithm is straight-forward, but it appears to be somewhat inefficient for use in material nonlinear analyses. This inefficiency arises from the need to decompose the stiffness matrix at every iteration. Since there are more efficient nonlinear solution algorithms available, it is probably better to perform nonlinear analyses in a code that has been optimized for better efficiency.

TEXGAP

This code was written by R. S. Dunham and E. B. Becker of the University of Texas for the Air Force Rocket Propulsion Laboratory, Edwards, California (Ref. 14). The code is currently restricted to linearly elastic, axisymmetric geometries; although non-axisymmetric loadings may be applied in the form of a Fourier series. TEXGAP represents a refined linear elastic computer code, and it has many unique features that make it very attractive for nozzle structural analysis. The code is undergoing continual development, and new features scheduled for addition are nonlinear small strain, large rotation behavior and three-dimensional analysis capability. The present relatively sparse user's manual is being updated and made more complete. The code is capable of static stress analysis only.

Elements

A. Current element library

1. QUAD 8 - A subparametric, 8-node (24DOF - 3DOF/node) quadrilateral element. The element employs quadratic displacement functions (linear strain) and may have isotropic or orthotropic material properties.
2. TRI - A reformulated, incompressible 6-node (21 DOF - 3 DOF/node plus 1 DOF/corner node incompressibility factor) isotropic triangular element. The element employs quadratic displacement functions (linear strain), and the mean (hydrostatic) pressure is represented by linear functions. The reformulated element is attributed to Dr. L. R. Herrmann, Univ. of California, Davis.

3. QUAD - A reformulated, isotropic 8-node (28DOF - 3DOF/node plus 1 DOF/corner node incompressibility factor) quadrilateral element composed of four TRI elements.

4. LINER - A thin (high aspect ratio) reformulated, isotropic 6-node (22DOF similar to QUAD element) quadrilateral element. A quadratic displacement field is employed for the long side while a linear field is employed along the short side. The element is designed for analysis of thin bond lines or rocket motor liner materials.

5. CASE - A thin, nonreformulated 6-node (18DOF - 3DOF/node) quadrilateral element. The element employs quadratic displacement fields, and materials may be isotropic or orthotropic.

6. CRACK - A special reformulated, isotropic element for calculating elastic type I and type II stress intensity factors. The element employs quadratic displacement functions on all sides and contains 11 nodes (33 DOF - 3 DOF/node).

B. Notes

1. TEXGAP has been constructed mainly for propellant and motor case analyses.

2. The element library needs to be expanded to include additional nonreformulated elements capable of isotropic, orthotropic, and anisotropic behavior for rocket nozzle analyses.

3. The elements may be used for plane stress/plane strain or axisymmetric solids of revolution.

Material Behavior

A. Current material models

1. Isotropic, incompressible behavior is considered for propellant elements.

2. Isotropic, plane orthotropic, and general orthotropic behavior is employed for CASE and QUAD 8 elements.

B. Notes

1. Only linear elastic materials are presently employed.

2. Thermal strains must be calculated externally to TEXGAP and input to the code.

Loading Conditions

A. Current loading conditions allowed

1. Boundary pressure loads and boundary shear stresses may be applied.

2. Uniform temperature change (not thermal gradients) may be applied.

B. Notes

1. Boundary loads may be skewed.
2. Both axisymmetric and nonaxisymmetric loads may be applied.
3. No provision for concentrated nodal loads.

Boundary Conditions

A. Current boundary conditions allowed

1. Nodal displacements may be specified.
2. Skewed displacement boundary conditions are allowed.

B. Notes

1. Nodal displacements may be nonzero.

Solution of Equations

A. Solution method

1. The frontal solution technique (attributed to B. M. Irons) is employed.
2. The number of nodes must be less than or equal to 1350. This means that the number of equations that can be solved is greater than 4000 (assuming 3DOF/node).

Special Features

A. Preprocessor

1. A mesh generator is available.
2. Serendipity-type polynomial functions are employed to map the internally generated meshes.

B. Postprocessing

1. Plot packages are available for CDC and UNIVAC computers.

C. Rezoning

1. A rezoning feature is available to allow one to zoom in on a particular region, regrid, and obtain a more detailed stress analysis.
2. The rezone feature employs boundary conditions obtained from the original coarse analysis.

D. Restart

1. This feature allows execution to be terminated at various points and restarted later.

E. The TEXGAP computer code is operational on CDC 6400 and UNIVAC 1108/1110 series computers.

Comments

TEXGAP has many features that make it attractive for nozzle stress analyses. It employs higher order elements (quadratic displacement fields), and several of the elements have unique characteristics; such as high aspect ratios, incompressibility, and cracks. The rezoning and restart features add to the code's versatility. However, there are some drawbacks in the code in its present configuration for application in nozzle analyses. The element library needs to be extended, and the capability for handling thermal gradients needs to be incorporated. Also, it would be advantageous to include temperature dependent material properties and material nonlinear behavior.

These drawbacks could easily be eliminated in future versions of the code, and then TEXGAP would become a very capable tool for both propellant grain and nozzle analyses.

NEPSAP

NEPSAP (Refs. 15, 16) is a production-oriented general purpose, nonlinear finite element code capable of large displacement thermoelastic-plastic and creep analysis of arbitrary structures. It was developed over the last four years by P. Sharifi and D. N. Yates, Lockheed Missiles and Space Company, Sunnyvale. The code is in full use at LMSC, and it should be considered a fully developed processor. The version of the NEPSAP code considered in this study was an old (1973) nonproprietary, static solution version available from ASIAC, Wright-Patterson AFB, Ohio. The present version at LMSC (October 1975) contains static and dynamic stress analysis capability, additional elements, and better solution algorithms; it is considered proprietary.

Elements

A. Current* element library

1. Beam-column elements - A three-dimensional, 2-node prismatic beam element with eccentricity option. It is capable of transmitting axial and bending loads (6DOF - 3DOF/node).
2. Two-dimensional continuum elements - A 4-node isoparametric element for two-dimensional plane stress/plane strain and axisymmetric solids (8DOF - 2DOF/node) is utilized.
3. Thin plate-shell elements - A fully compatible plate bending (Clough - Felippa quadrilateral) element composed of four linear curvature triangles (24DOF - 3DOF/node) is available. The linear curvature triangular element is also available.
4. Thick-shell element - A three-dimensional, 16-node isoparametric thick-shell element (48DOF - 3DOF/node) is employed.
5. Solid element - A three-dimensional, 8-node isoparametric brick element (24DOF - 3DOF/node) is available.

B. Notes

1. The thick-shell element is a degenerate 20-node isoparametric element. The displacement functions are linear through the thickness and quadratic on the upper and lower surfaces.

Material Behavior

A. Current material models

1. Isotropic and orthotropic elastic behavior is considered.
2. Anisotropic, elastic multi-layered thin-plate behavior is employed.
3. Incremental thermoelastic-plastic behavior is available, and either isotropic or kinematic (with Ziegler's modification) hardening models are utilized. Either linear or piecewise-linear hardening behavior may be used, but only isotropic material behavior is allowed in plasticity and creep solutions.
4. Creep and relaxation models employ the Norton-Odqvist stationary-creep power law.
5. Temperature dependent material properties may be utilized with all elements.
6. Fully general, large displacement strain relations are employed allowing large geometric deformations to be solved.

B. Notes

1. Material properties are input in tabular form; no built-in properties are available.

*Current to ASIAC Version

2. Orthotropic materials are not available for creep and plasticity models.

3. Elastic-plastic and creep material models are based on small strain theories where superposition of elastic and plastic strains is valid.

Loading Conditions

A. Current loading conditions available

1. Thermal loads are utilized.

2. Distributed pressure loads are employed. Only normal pressures are available for solid elements and thick-shell elements. Tangential pressures (surface shears) are available on axisymmetric and thin plate/shell elements.

3. Concentrated nodal loads may be applied.

4. Non-conservative loads may be utilized.

B. Notes

1. There are apparently no provisions for distributed body force loadings, such as inertial loads. This type of loading would have to be generated externally and input as concentrated nodal loads.

2. Concentrated nodal loads may be skewed.

Boundary Conditions

A. Current capabilities

1. Each node point contains six degrees-of-freedom, three translations and three rotations.

2. Any combination of nodal point freedoms may be set to zero.

3. Enforced displacements (nonzero) may be applied to any node.

B. Notes

1. Nodal point displacements may be skewed.

2. The enforced displacement algorithm may not be working on the ASIAC version of NEPSAP.

Solution of Equations

A. Current capability

1. An incremental nonlinear solution algorithm is employed.

2. The program can handle 3000 nodes (6DOF/node) in a 65K core computer; no limitation on number of elements or bandwidth.

3. No description of the solution algorithm is given in user's manual.

4. Provisions for equilibrium correction are available for improved accuracy.

5. A total Langrangian incremental formulation is used for the equilibrium equations.

B. Notes

1. LMSC personnel have installed better, faster algorithms on the proprietary version of NEPSAP.

2. Calculations for static stresses (nonlinear) are carried out in single precision.

3. The first increment of a nonlinear analysis is the linear elastic solution, so NEPSAP may be used for linear analysis also.

Special Features

A. Preprocessor

1. A preprocessor program is available to generate nodes, elements, loads, and boundary conditions.

2. The preprocessor requires a small user-written program to call the desired subroutines.

3. Card-by-card input is available.

B. Postprocessor

1. A postprocessor (graphics package) is available for UNIVAC computers.

C. Buckling analysis under non-conservative loading is available.

D. Restart capability is available.

E. The NEPSAP computer code is available from ASIAC for CDC computers. A UNIVAC version of ASIAC-NEPSAP is currently being debugged at NWC, China Lake. These two codes are nonproprietary. LMSC has proprietary versions of NEPSAP running on UNIVAC and CDC computers. An IBM 360 version of LMSC-NEPSAP is being debugged at Sunnyvale.

Comments

The NEPSAP computer code currently contains more of the features desired for nozzle analysis than any of the codes surveyed. Personnel at LMSC are presently adding dynamic analysis and heat transfer analysis capabilities to

NEPSAP. The heat transfer analysis capability will be a stand alone processor that is fully compatible with the NEPSAP preprocessor and processor. Hopefully, these improvements to NEPSAP will be brought into the public domain at some later date.

The present nonproprietary version of NEPSAP is certainly adequate to perform analyses of the type currently being conducted in the propulsion community. Drawbacks normally associated with large, general purpose computer codes vs. small special purpose codes will be qualitatively evaluated during the present study, and conclusions on this will be forthcoming.

NONSAP

NONSAP (Refs. 17, 18) is a research-oriented finite element computer code for the linear and nonlinear, static and dynamic analysis of structures. The code was primarily developed by Dr. K. J. Bathe at the University of California, Berkeley, under the direction of Dr. E. L. Wilson. The code has limited general structural analysis capability since one-, two- and three-dimensional elements are available; however, only the two-dimensional element material library is fairly complete.

Elements

A. Current element library

1. One-dimensional rod element - A 2-node (6DOF - 3DOF/node) axial bar type element for use in three-dimensional truss analyses is available.
2. Two-dimensional continuum elements - A variable-number-node (4- to 8-node) isoparametric element is available for two-dimensional plane stress/plane strain and axisymmetric solids of revolution. The element has 8 to 16DOF (2DOF/node), and is therefore only capable of transmitting axisymmetric loads.
3. Three-dimensional solid or thick-shell element - A variable-number-node (8- to 21-node) isoparametric element is employed for three-dimensional analysis. The element has 24 to 63DOF depending on how it is utilized (3DOF/node).

B. Notes

1. The two- and three-dimensional elements can be tailored so that low-order elements are used in low strain gradient regions and high-order elements are used in high strain gradient regions.

2. It is possible to construct transition elements between low and high strain regions by a proper input element choice.

Material Behavior

A. Current material models

1. Isotropic linear and nonlinear elastic models are available for truss-type and three-dimensional solid elements.
2. Several material models are available for two-dimensional elements.
   a. Linear elastic, isotropic and orthotropic materials are utilized.
   b. Curve description models are employed, including multilinear segments and tangent modulus descriptions.
   c. Elastic-plastic models are available utilizing either Drucker-Prager or Von Mises Yield conditions.
   d. An incompressible, isotropic elastic (Mooney-Rivlin) material model is utilized for plane stress analyses only.

B. Notes

1. Truss elements may have linear, material nonlinear, or small strain-large rotation behavior.
2. Two-dimensional elements may employ linear, material nonlinear, total Langrangian, or updated Langrangian formulations.
3. Three-dimensional elements may employ linear or material nonlinear behavior only.
4. User-supplied material models may be employed.

Loading Conditions

A. Current loading conditions

1. Only concentrated nodal loads may be specified.

B. Notes

1. Distributed pressures, body loads, and thermal loads must be generated externally and input as concentrated nodal loads.

Boundary Conditions

A. Current boundary conditions

1. Element nodes can be either fixed or free in translation only.
2. No provision for enforced displacements is available, although a method of employing truss-type elements to specify boundary displacements is described in the user's manual.

B. Notes

1. Skewed boundary displacements are not allowed.

Solution of Equations

A. Current capability

1. The equilibrium equations are formulated in incremental form based on either a total Langrangian or updated Langrangian coordinate system.
2. Static and dynamic analysis capability is available.
   a. Static analysis is performed using a Guassian elimination "skyline" scheme for efficiency.
   b. Dynamic analysis can be performed using Wilson - θ or Newmark β time integration schemes.
   c. The eigensystem solution scheme in NONSAP employs the determinant search method.

B. Notes

1. The equilibrium equations are solved in-core; it is not apparent on a first reading of the user's manual what maximum number of equations is allowed.

Special Features

A. There is no preprocessing or postprocessing available.

B. Restart capability is provided.

C. NONSAP is written in FORTRAN IV for the CDC 6400 computer; however the program has been successfully compiled on IBM 360 and UNIVAC 1110 computers. Execution of the sample problems on the NWC UNIVAC 1110 computer gave incorrect results for the dynamic solutions; however the static solutions were correct.

Comments

NONSAP contains some of the most highly developed features of the four selected codes. It contains many capabilities not found in the other codes, and has a fairly complete two-dimensional element library. Since NONSAP was designed as a research tool, it has not been optimized for solving large, production problems. The lack of preprocessing and postprocessing might hinder the use of NONSAP in a production environment. On the other hand, NONSAP is well written and organized so that the program may be readily expanded at the user's option.

SECTION V

CONCLUSIONS

Several computer codes were surveyed to determine their applicability for nozzle stress analysis. These codes included linear and nonlinear stress analysis capabilities. Four codes, SAAS III, TEXGAP, NEPSAP, and NONSAP were selected for further evaluation. These codes will be installed on the CDC 6400 computer at the Air Force Rocket Propulsion Laboratory, Edwards AFB, California, and evaluated by comparison on four sample problems.

A summary of all the codes evaluated on this study is given in Appendix A. Four sample problems acquired for use in the comparison are summarized in Appendix B.

This report documents the first part of this study on nonlinear nozzle structural analysis computer codes; namely, the evaluation and selection of candidate codes for nozzle stress analysis, and the definition of features desired in these codes. A second report will document the procedures needed to implement the codes on the AFRPL CDC 6400 computer and present the comparison results for the sample problems. A summary of deficiencies found in current rocket nozzle stress analysis codes will be given in the second report.

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APPENDIX A

SUMMARIES OF CODES EVALUATED IN THIS STUDY

This appendix contains capsule summaries of the computer codes considered for evaluation in this study.

I. Two-Dimensional Axisymmetric Codes

- A. GNATS
- B. PLACID
- C. CREEP-PLAST
- D. ASAAS
- E. DOASIS

II. General 3-D Codes

- A. SAP IV
- B. NASTRAN
- C. VISCEL

GNATS

GNATS (Refs. 19, 20) is a system of computer codes containing a preprocessor, processor, and postprocessor, designed for nonlinear analysis of two-dimensional and axisymmetric structures. The routines were developed at Sandia Laboratories, Livermore, by R. C. Young and M. L. Callabresi. The program is available for UNIVAC and CDC computers.

Elements

Either 4- or 8-node isoparametric, quadrilateral elements are available.

Material Behavior

Both material and kinematic nonlinearities may be employed, but only isotropic material behavior is allowed. Isotropic or kinematic hardening may be employed, and either Ramberg-Osgood or multilinear segment stress-strain descriptions may be utilized.

Loading Conditions

Acceleration loads, distributed pressures and shears, and follower forces may be applied. Thermal loads may be input by specifying a temperature distribution. The complete load path must be defined including loading, unloading, heating, cooling, and pressurization.

Boundary Conditions

Concentrated nodal forces and boundary displacements may be specified. Skewed forces and displacements are allowed.

Solution of Equations

An incremental solution based on the total Lagrangian equation formulation is used in GNATS. The user has general control over the solution by specifying the number of load increments and convergence criteria to be used in each problem. The problem size is limited only by the available storage in a given computer. Since dynamic storage allocation is employed, each user must determine the limits of his own computer.

Special Features

A Laplacian mesh generation scheme is employed and extensive postprocessing is available. Material data may be input in Lagrangian, Eulerian, or mixed coordinate systems, and several different materials may be utilized.

Comments

The GNATS code is well-documented and contains many good features. It is capable of handling almost any loading condition that could arise. The main drawback in the program currently is the lack of material models to characterize orthotropic and anisotropic materials. There are plans to incorporate additional material models into GNATS, but it is not known when these additions will be made available.

PLACID

PLACID (Ref. 22) is general nonlinear computer code for stress analysis of two-dimensional and axisymmetric structures. The code was written by R. G. Lawton of Los Alamos Scientific Laboratory, Los Alamos, New Mexico. The code is operational on a CDC 6600/7600 computer system at LASL.

Elements

The basic element in PLACID is the well-known constant strain triangle. Quadrilateral elements and higher order elements with up to six sides may also be constructed using several triangular elements.

Material Behavior

Material properties may have complete three-dimensional anisotropy, time and temperature dependence, and different properties in tension and compression. Only material nonlinearities are considered.

Loading Conditions

Body forces (force per unit volume) may be applied.

Boundary Conditions

Boundary forces and displacements may be applied. Boundary displacements may be skewed, but forces must be input in the global X-Y coordinate system.

Solution of Equations

Gaussian elimination with back substitution is employed, however the elimination is carried out as soon as an equation is completed to reduce storage requirements. The equation solution is nested within two DO loops that control the increments of loading and iterations per load increment. The maximum size problem that can be solved is determined by the storage available at a given facility. Dynamic storage allocation is employed.

Special Features

All input data is programmed via subroutines supplied by the user. There are no READ statements employed in the main program.

Comments

This code seems to be more of a research tool than a production oriented code. It has capabilities for solving very large problems with very complex material behavior; however, the lack of preprocessing and postprocessing are drawbacks.

CREEP-PLAST

CREEP-PLAST (Refs. 23, 24) is a computer code for analysis of two-dimensional and axisymmetric structures undergoing simultaneous creep and plastic deformations. The code was written by Y. R. Rashid, General Electric Co., San Jose for Oak Ridge National Laboratory. The program is operational on the IBM 360 computer system.

Elements

Triangular and quadrilateral elements are available. These elements are probably of the constant strain type; however, neither of the above references explicitly state this.

Material Behavior

Isotropic materials employing instantaneous time-independent elastic-plastic and time-dependent creep are utilized. Kinematic hardening is used with the Von Mises yield condition and its associated flow law. The creep formulation employs the equation-of-state-theory or a hereditary integral-type theory. Only material nonlinearities are considered.

Loading Conditions

Complete load histories for both thermal loads and concentrated nodal loads must be input. Distributed pressures may be applied.

Boundary Conditions

Specific boundary displacements may be applied but they must remain constant throughout the solution. The specified displacements may be skewed.

Solution of Equations

An incremental formulation of the nonlinear system of equations is employed. The code is currently restricted to 900 nodal points (1800 equations) and 1800 elements. Considerable data is given in the user's manual for estimating run times.

Special Features

The latest LMFBR creep equations for 304 stainless steel are incorporated into the program.

Comments

This code is specialized to compute stresses in LMFBR vessels, components, and core structures; it does not appear to be too useful for application in rocket nozzle analyses.

ASAAS

ASAAS (Ref. 25) is similar to the SAAS III code except that it was developed to solve axisymmetric structures undergoing non-axisymmetric loading. Material properties may vary circumferentially. The code was developed by J. G. Crose, Aerospace Corporation, San Bernardino. The similarities between ASAAS and SAAS III preclude the necessity for detailed descriptions of ASAAS capabilities.

DOASIS

DOASIS (Refs. 30, 31, 32) is a computer code for stress analysis of two-dimensional and axisymmetric, orthotropic structures. The code was written by F. C. Weiler of Weiler Research Inc., Mountain View, California.

Elements

DOASIS employs both a constant strain triangle and a quadrilateral made from four CST triangles.

Material Behavior

Orthotropic material behavior is available. A nonlinear, multimodulus deformation plasticity model is employed. Material properties may be different in tension and compression, and temperature dependence is allowed. Geometric nonlinearities are not allowed.

Loading Conditions

Acceleration loads, thermal loads, boundary pressures, boundary shear stresses, initial stresses, and initial strains may be applied. Concentrated nodal forces may also be employed.

Boundary Conditions

Specified boundary displacements may be applied; boundary displacements may be skewed. There are provisions for interference fit boundary conditions.

Solution of Equations

Gaussian elimination is employed in the DOASIS equation solver. An iterative technique is incorporated to solve for nonlinear effects. The stiffness matrix bandwidth is the only factor affecting the maximum size problem that can be solved. Dynamic storage allocation is employed with the solution spilling over to auxiliary storage if required.

Special Features

Preprocessing and postprocessing are available with the computer code. Both a residual error and an equilibrium check are made to aid in assessing solution accuracy.

Comments

The manuals describing DOASIS (Refs. 30, 31, 32) are extremely well written, and the code is very completely documented. Additionally, considerable background information is contained in these references. DOASIS was not available for evaluation at the time when the codes were being acquired for this study, otherwise it would have surely been included in this study. The code contains many features desirable for nonlinear nozzle structural analysis. The only major deficiency is the inability to solve large geometric deformations.

SAP IV

SAP IV (Ref. 26) is a general structural analysis code for one-, two- and three-dimensional structures. It was written by K. J. Bathe, E. L. Wilson, and F. E. Peterson of the University of California, Berkeley.

Elements

The element library includes a one-dimensional truss element; a three-dimensional beam element; two-dimensional plane stress, plane strain, and axisymmetric elements; a three-dimensional solid element; thick and thin shell elements; and a pipe element. The axisymmetric elements may be either triangular or quadrilateral; they are based on a linear displacement field, isoparametric formulation.

Material Behavior

Only linear elastic behavior is considered. Materials may be isotropic or orthotropic, and the properties may be temperature dependent.

Loading Conditions

Gravity loads, thermal loads, and distributed surface tractions may be applied. Concentrated nodal loads may also be employed.

Boundary Conditions

Zero boundary displacements may be easily applied; enforced boundary displacements must be applied by using some "trickery" as explained in the manual (special boundary elements are employed).

Solution of Equations

Both static and dynamic analyses may be performed with SAP IV. Eigenvalue analysis, transient response analyses, and response spectrum analyses may be performed.

Special Features

SAP IV consists only of a processor program. No standard preprocessor or postprocessor is available with the code; however, several pre and postprocessors are available from various users of the code.

Comments

SAP IV was used as the basis to develop NEPSAP, and while the ASIAC version of NEPSAP is for static solutions only, later proprietary versions of NEPSAP include dynamics capabilities. Therefore, SAP IV provides a linear elastic, dynamic capability for general problems that are not included in the ASIAC-NEPSAP code. Later versions of NEPSAP should do everything SAP IV will do and provide nonlinear static and dynamic analysis capabilities also; however, SAP IV is a fairly efficient, versatile processor code for linear elastic analyses.

NASTRAN

NASTRAN (Refs. 23, 28, 29) has already been widely documented. Only a few highlights of its capabilities are given here for axisymmetric structural analysis. NASTRAN (level 15.5) employs triangular and quadrilateral axisymmetric elements. The elements are basically the same constant strain-type elements employed in SAAS III and DOASIS. Element loadings include distributed pressures, body forces, thermal loads, and concentrated loads.

Fluid elements may be coupled with axisymmetric elements to solve fluid-structure interaction problems. Both static and dynamic analyses may be performed.

VISCEL

VISCEL (Ref. 21) is a general one-, two-, and three dimensional structural analysis code. The code was written by K. K. Gupta, F. A. Akyuz, and E. Heer of the Jet Propulsion Laboratory, Pasadena. VISCEL was written especially to solve linear, thermoviscoelastic structural response problems. The solution is assumed to be quasi-static, and thermorheologically simple material behavior is assumed. Material behavior is modeled by completely general (small strain) anisotropic relaxation moduli having up to 21 independent components. A finite element formulation is employed and a special incremental solution technique is utilized.

This code was written especially for propellants and similar materials requiring a viscoelastic characterization of their properties. VISCEL probably would not be too useful for nozzle material analysis at this point in time, since we are still heavily involved in understanding the linear elastic behavior of these materials. If the necessity should arise, however, that viscoelasticity has to be considered for determining high temperature behavior of nozzle materials, then VISCEL or a code like it would be necessary for proper analyses.

APPENDIX B

SUMMARIES OF SAMPLE PROBLEMS FOR THIS STUDY

I. Plane Strain Cylinder

II. Axisymmetric Sphere

III. Axisymmetric Cylinder

IV. Axisymmetric Nozzle Thr oat Components

SUMMARY OF SAMPLE PROBLEMS FOR THIS STUDY

The sample problems employed in this study were selected to provide a basis for comparing the various computer codes being evaluated. The first three sample problems were chosen because the propulsion community had previous experience with them. In December 1973, a set of three stress analysis problems was sent to all solid propellant missile contractors by the JANNAF Structures and Mechanical Behavior Working Group and the Operational Serviceability Working Group. The purpose of these problems was to compare finite element capability between contractors.

Since many of the propulsion contractors have solved these problems on their in-house computing facilities, they should be able to obtain a comparison between the present codes and their own codes. The three problems used in this study are as follow:

Problem No. 1 - Plane Stress Cylinder Under Internal Pressure

This problem is fully specified in Figure B-1. The desired quadrilateral grid structure has element boundaries with radii of: 1.0, 1.2, 1.5, 1.9, 2.4, 3.0, 3.7, and 4.5 inches; the grid spacing in the circumferential direction is a uniform 10 degrees. Symmetry boundary conditions are specified along the vertical and horizontal axes.

Four solution parameters are desired as a function of radial position: (1) the radial displacement (i.e., the "y" displacement along the y axis or the "x" displacement along the x axis (they should be identical); (2) the hoop strain (i.e., the maximum principal strain in the x-y plane, along any one element ray); (3) the radial stress (i.e., the minimum principal stress in the x-y plane, along any one element ray); and (4) the hoop stress (i.e., the maximum principal stress in the x-y plane, along any one element ray).

The exact solution for this problem is given by:

$$u = \frac{a^2 P_i}{\left(b^2 - a^2\right) E} \left[(1 - \nu)\, r + (1 + \nu) \frac{b^2}{r}\right]$$

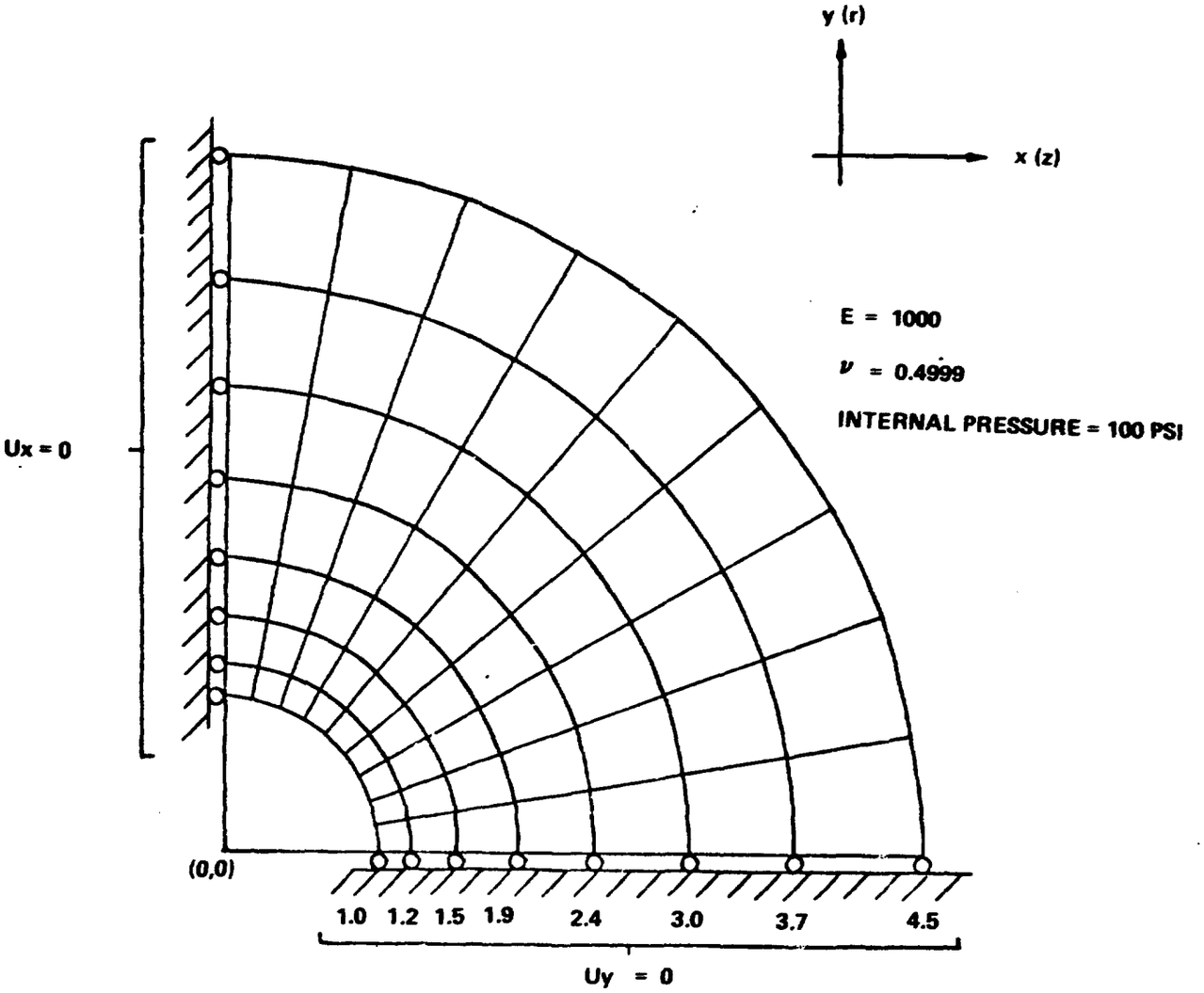

Figure B-1. Plane-Stress Cylinder Quadrant Under Internal Pressure Loading

$$\epsilon_\theta = \frac{u}{r}$$

$$\sigma_r = \frac{a^2 P_i}{b^2 - a^2}\left(1 - \frac{b^2}{r^2}\right)$$

$$\sigma_\theta = \frac{a^2 P_i}{b^2 - a^2}\left(1 + \frac{b^2}{r^2}\right)$$

Problem No. 2 - Sphere Under Internal Pressure

The grid for this problem is identical to that for Problem No. 1, Figure B-1. However, the axis of revolution is the x axis (i.e., $u_r = 0$ along this line, as well as $u_z = 0$ along the vertical axis). For this check problem, the radial displacement, the hoop strain, the radial stress, and the hoop stress as a function of the radial position in the sphere are desired.

The exact solution for this problem is:

$$\sigma_r = P_i \frac{a^3\left(b^3 - r^3\right)}{r^3\left(a^3 - b^3\right)}$$

$$\sigma_\theta = P_i \frac{a^3}{2r^3} \frac{\left(2r^3 + b^3\right)}{\left(b^3 - a^3\right)}$$

$$\epsilon_\theta = \frac{1}{E}\left[\sigma_\theta - \upsilon\left(\sigma_\theta + \sigma_r\right)\right]$$

$$u = r\epsilon_\theta$$

Problem No. 3 - Finite-Length Cylinder Under Shrinkage

[redacted] problem is specified in Figure B-2. The desired axial grid spacing is given adjacent to the horizontal grid line nearest the axis, and the radial grid spacing is given adjacent to the left vertical grid line. The r and z displacement components are fixed to zero along the outer cylinder circumference. The

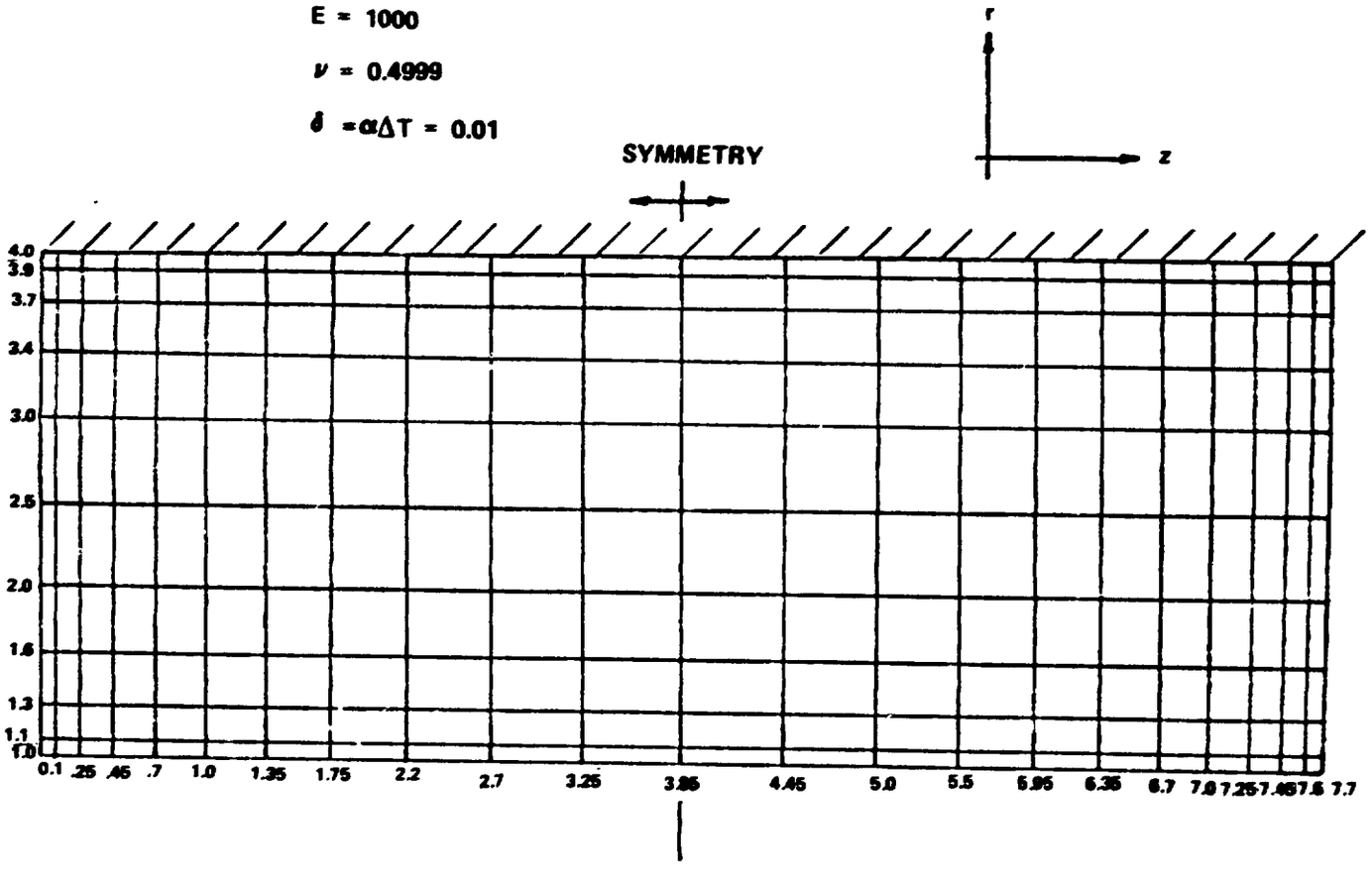

Figure B-2. Finite-Length Cylinder Under Shrinkage

grid structure has symmetry about the axial mid-plane. It is intended that this plane of symmetry be used to assess the numerical (round-off-error) accuracy of the finite-element program. However, because of long run times it might be necessary to use only one-half of the grid structure.

The normal displacement along the free surface boundary, as a function of arc length from the left upper grid corner is desired. Thus, the normal displacement will be in the "z" direction for the first 3.0 inches of arc length, and in the "r" direction for the next 3.85 inches of arc length.

The calculated values for the axial strain, hoop strain, axial stress, and hoop stress along the element row nearest the centerport are also desired. The strain values desired are the "total" strains, not the "stress producing" strains; i.e., $\epsilon_T = \bar{\epsilon} - \delta$, where ϵ_T is the total strain, $\bar{\epsilon}$ is the stress producing strain, and δ is the shrinkage coefficient ($\delta = \alpha\Delta T$).

Problem No. 4 - Throat Insert Cooldown

The fourth sample problem was chosen to evaluate the selected computer codes on a typical rocket nozzle problem. This problem was taken from an analytical study[1] conducted by TRW Systems Group, Redondo Beach for AFRPL.

Initially, it was desired to evaluate a material nonlinear solution with a thermal gradient input. Various deficiencies in each of the codes precluded obtaining the desired solution. The SAAS III code came closest to satisfying the needs, but the nonlinear, orthotropic material model in SAAS was suspect. TEXGAP could solve only linear elastic, orthotropic materials with a uniform step change in temperature, while ASIAC-NEPSAP did not contain an orthotropic, axisymmetric element at all. A linear elastic, orthotropic material model was added to ASIAC-NEPSAP so that we could attempt this sample problem. The nonlinear, isotropic axisymmetric element in ASIAC-NEPSAP was not affected by our modification. NONSAP contained a linear elastic, orthotropic element, but thermal loads were not included. It was decided not to try to incorporate thermal loads into NONSAP at this time due to funding and time limitations.

[1] King, K. R., "Nonlinear Stress Analysis of Codeposited Silicon Carbide-Pyrolytic Graphite Coated Rocket Nozzle Throat Inserts," Final Report to Atlantic Research Corporation, Cont. No. P.O. 78916, TRW Systems Group, Redondo Beach, Calif., Aug. 28, 1973.

Finally, in order to compare as many codes as possible, it was decided to restrict the sample problem to a linear elastic solution with a uniform step temperature change. It was hoped that the nonlinear problem with a thermal gradient would be attempted at a later date.

The fourth sample problem consisted of a throat insert made of 20 percent silicon carbide-pyrolytic graphite codeposited on an ATJ substrate. The finite element mesh used in the analysis is shown in Figure B-3. The throat diameter was taken as 1.72 inches, while the insert O.D. was 3.0 inches. The coating thickness varied from 138 mils at the entrance end to 139 mils at the exit end of the insert.

Material properties used in this analysis were taken from the aforementioned TRW report. These properties are summarized in Table B-1, Table B-2, and Figures B-4 through B-9.

The nozzle components were assumed to cool down from 3200°F to room temperature (taken as 70°F). The insert was restrained only along the left edge (entrance plane in the axial direction only) to prevent any possible rigid body motion.

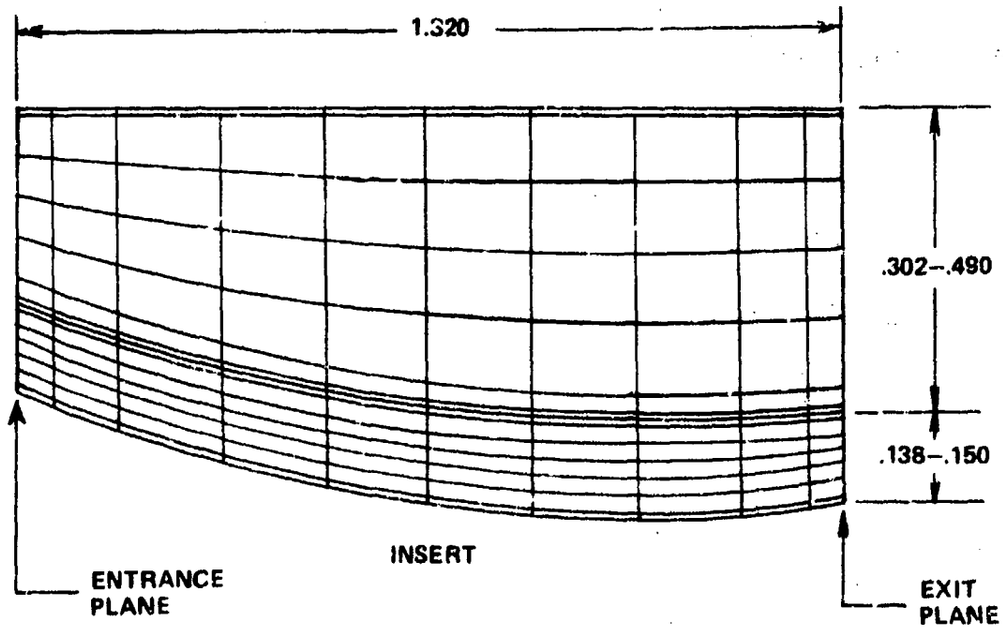

Figure B-3. Finite Element Mesh, ARC 1.72 Inch Nozzle 0004

Table B-1. Mechanical Properties - PG-20% SiC

Temp. °F	E 10^6 psi	σ_y psi	n	ν	G 10^6 psi
70	4.7	24000	.333	.14	2.06
500	4.9	24000	.353	.14	2.15
1000	5.1	24000	.373	.14	2.24
1500	5.3	24000	.393	.14	2.32
2000	5.5	24000	.413	.14	2.41
2500	5.7	24000	.433	.14	2.50
3000	5.8	24000	.443	.14	2.54
3500	5.4	22000	.415	.14	2.37
4000	4.2	17000	.333	.14	1.85
4500	2.4	5000	.333	.14	1.06
5000	1.4	3000	.333	.14	.61

Table B-2. Mechanical Properties - ATJ

Temp. °F	E_{wg} 10^6 psi	E_{ag} 10^6 psi	$\sigma_{y_{wg}}$ psi	$\sigma_{y_{ag}}$ psi	n	ν_{w-w}	ν_{a-w}	G_{w-a} 10^6 psi
70	1.1	.8	4500	4100	.333	.14	.10	.418
500	1.2	.85	4620	4100	.31	.15	.113	.443
1000	1.31	.9	4760	4100	.28	.16	.127	.466
1500	1.42	.95	4900	4100	.25	.17	.14	.49
2000	1.51	1.0	5100	4100	.22	.18	.153	.51
2500	1.59	1.04	5300	4100	.20	.19	.167	.524
3000	1.65	1.06	5300	4200	.20	.20	.18	.53
3500	1.62	1.04	5300	4300	.20	.20	.19	.514
4000	1.42	.95	4900	4000	.30	.20	.20	.456
4500	.9	.75	3000	2500	.20	.20	.20	.333

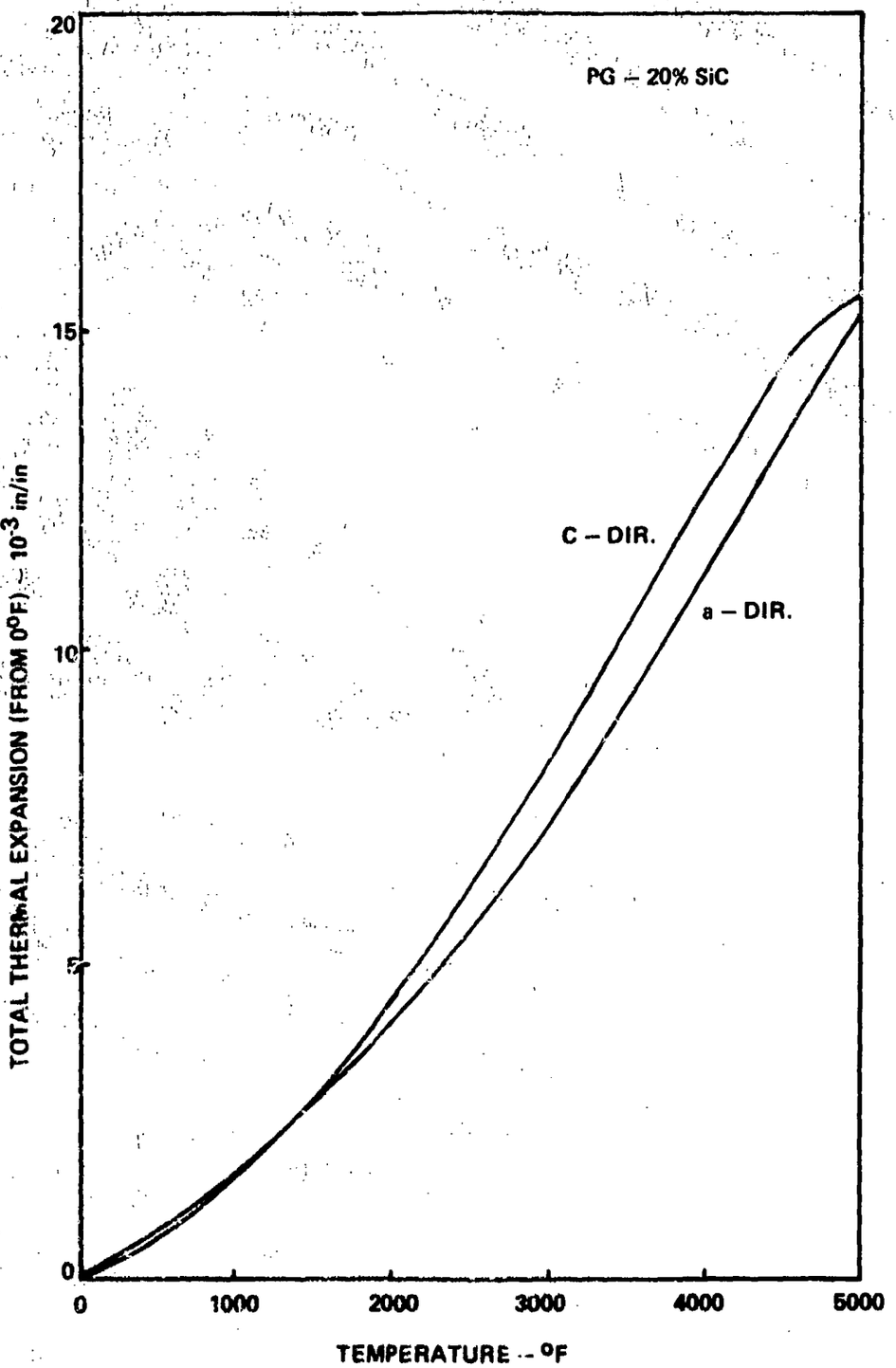

Figure B-4. Thermal Expansion, PG-20% SiC

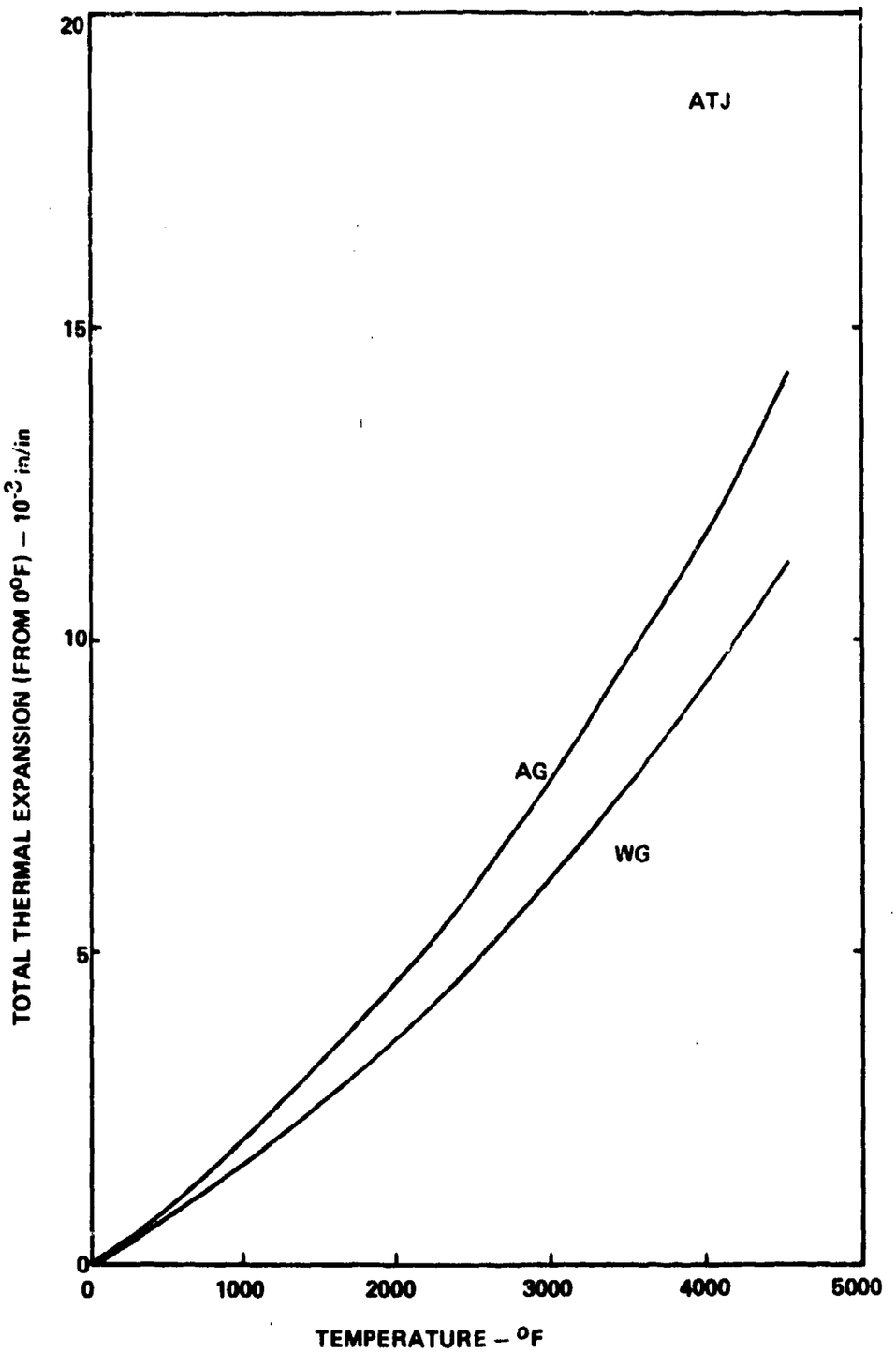

Figure B-5. Thermal Expansion, ATJ Graphite

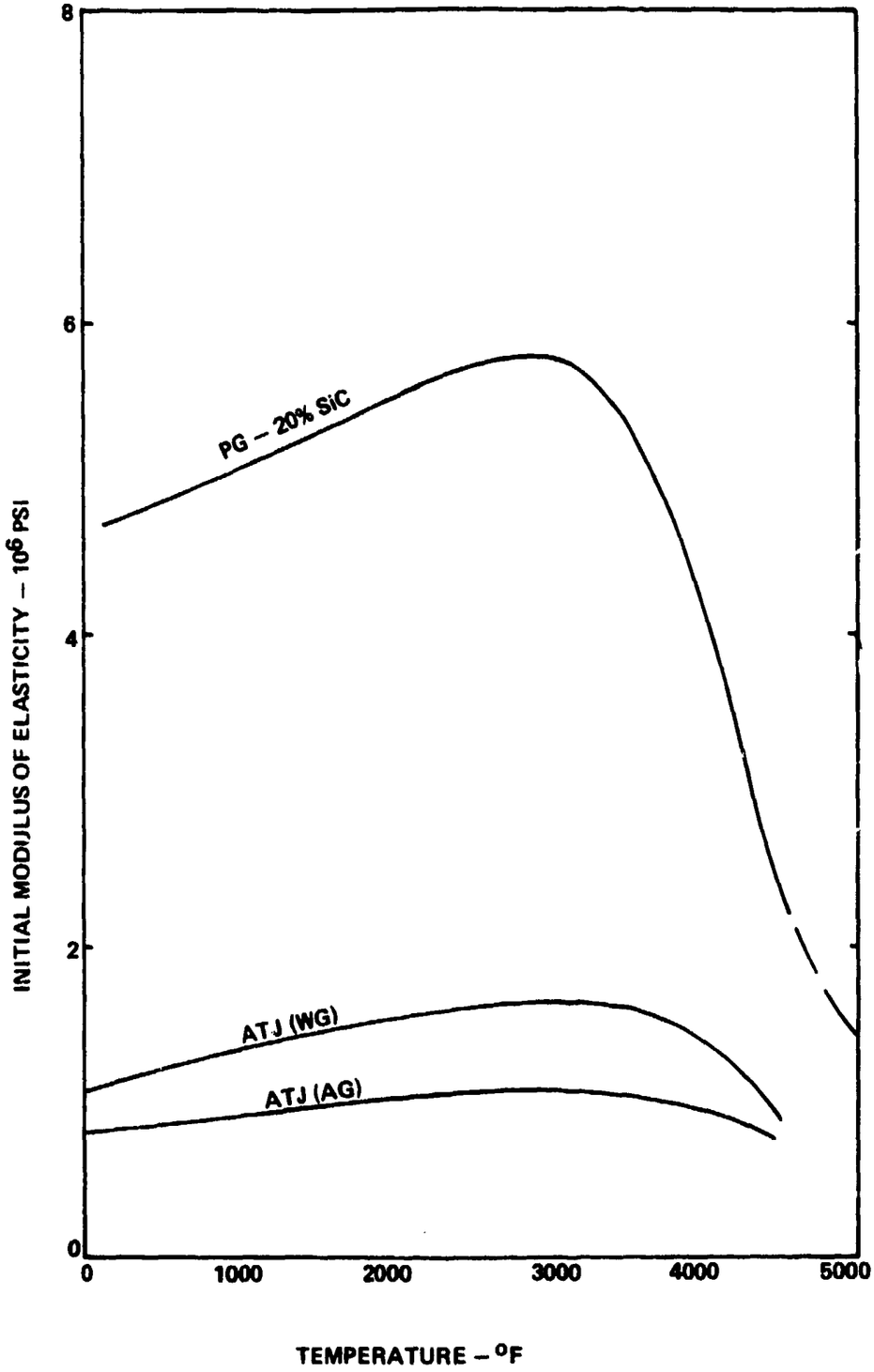

Figure B-6. Initial Modulus of Elasticity, PG-20% SiC and ATJ Graphite

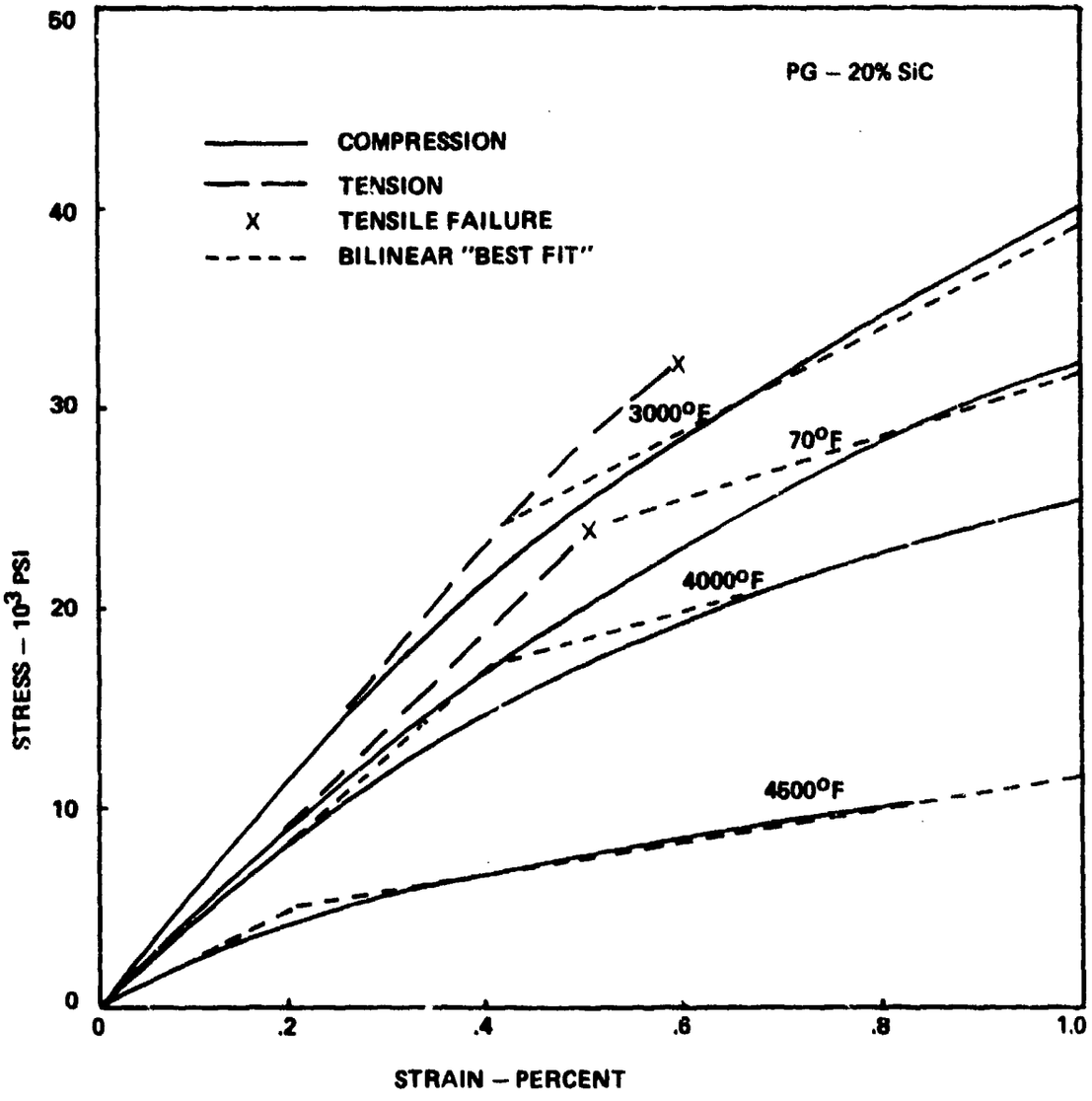

Figure B-7. Uniaxial Stress-Strain Curves, PG-20% SiC

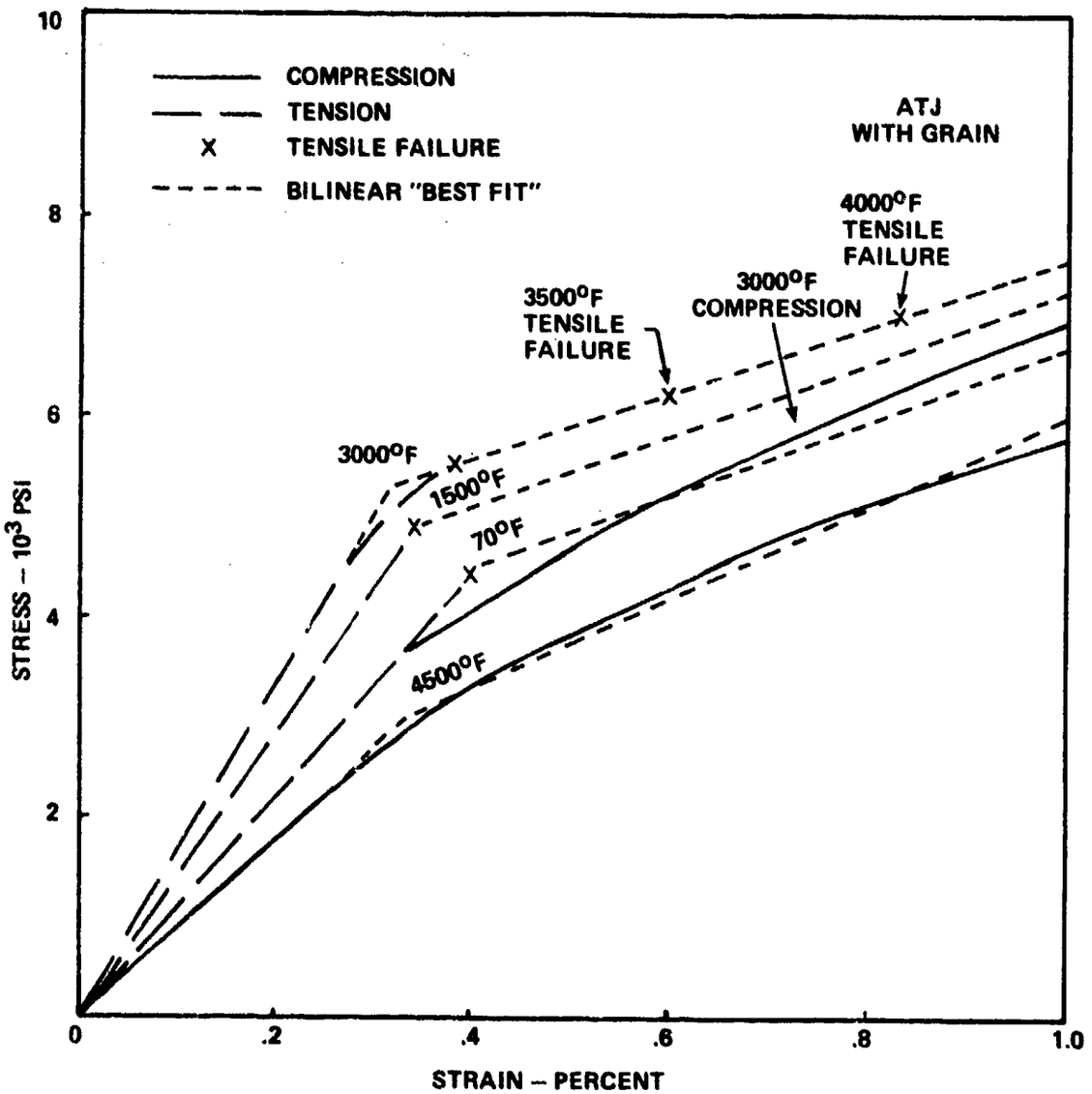

Figure B-8. Uniaxial Stress-Strain Curves, ATJ Graphite, With Grain

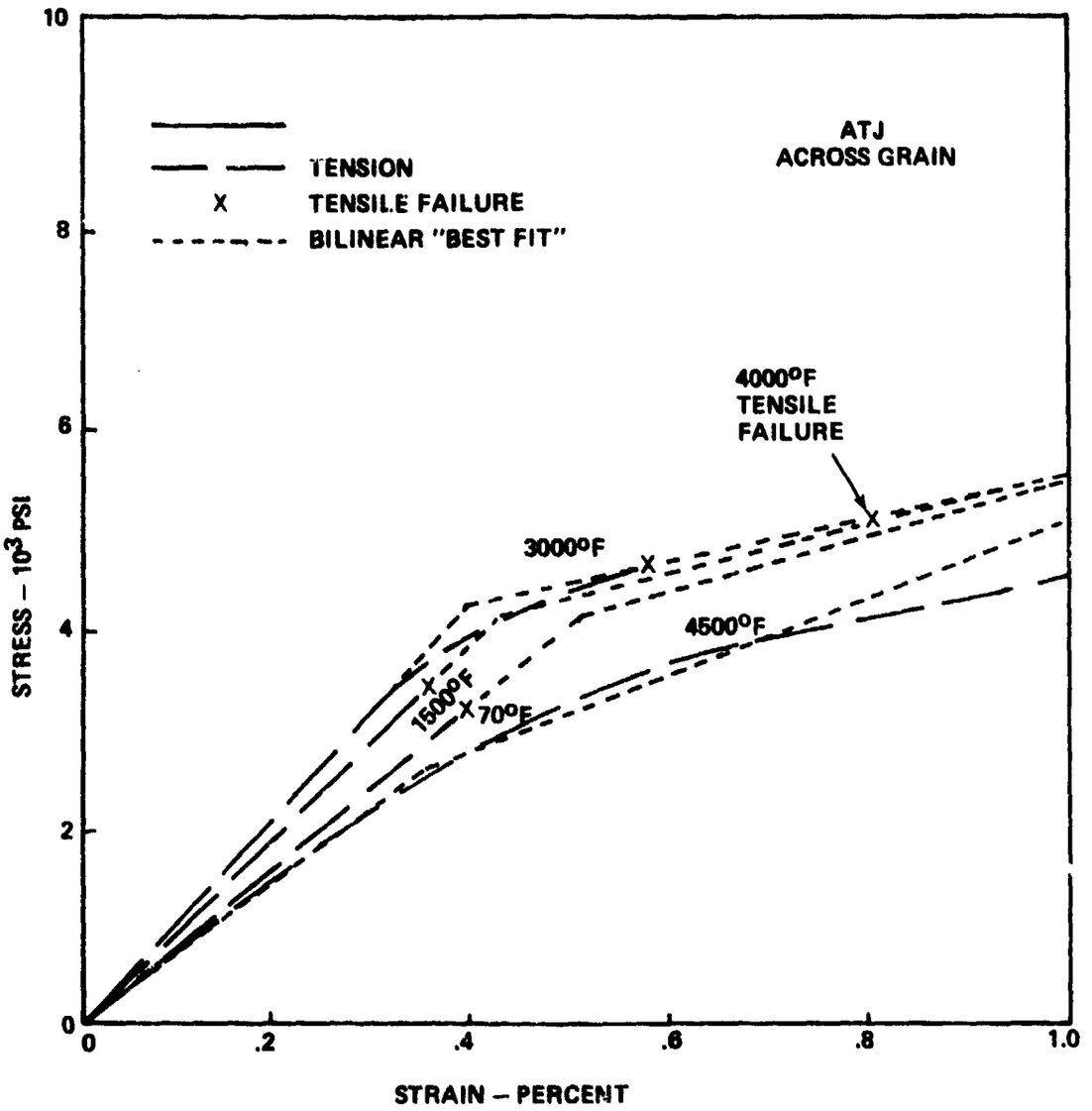

Figure B-9. Uniaxial Stress- Strain Curves, ATJ Graphite, Across Grain